Migration Operator Compression by Wavelet Transform: Beamlet Migrator

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Summary

We study one-way wave propagation in the frequency-wavelet domain (frequency-beamlet domain) for different wavelet bases, and compare their performances for migration/imaging. We show that the Kirchhoff operator in space domain is a dense matrix, while the compressed beamlet-propagator matrix which is the wavelet decomposition of the Kirchhoff operator (or other one-way wave propagators), is a highly sparse matrix. For sharp and short bases, such as the Daubechies 4 (D4), both the 'interscale and intrascale coupling are strong. However, the interscale coupling is relatively weak for smooth bases, such as higher-order Daubechies wavelets, Coiflets, and spline wavelets. The images obtained by the compressed beamlet operator are almost identical to the images from a full-aperture Kirchhoff migration. Compared with the limited aperture Kirchhoff migration, beamlet migration can obtain much wider effective apertures, hence higher resolution and image quality, with similar computational efficiency. The compression ratio of the migrator ranges from a few times to a few hundred times, depending on the frequency, step length and the wavelet basis. Combining with the data (wavefield) compression, even greater efficiency can be expected.

Introduction

Seismic data compression has achieved remarkable success. However, at present data compression is focused on data transportation and storage problems, and is assumed to be transparent to processing. Little has been done towards directly processing compressed data in the wavelet domain and, furthermore, compressing the processing operators themselves. Nevertheless some progress has been reported in the application of wavelet transform to wave propagation and migration (Le Bras and Mellman, 1994; Dessing and Wapenaar, 1995; Mosher and Foster, 1995; Wu and McMechan, 1995; Mosher et al., 1996; Wang and Pann, 1996). Our approach is to study both data compression and migration operator compression using wavelet transform. This approach requires much dedicated theoretical study on wavelet transform and wave propagation. Operator compression may have quite different requirements on the properties of wavelet bases. Seismic data compression is intended

to find efficient ways of representing data. Yet David Marr reminded us, "how information is presented can greatly affect how easy it is to do different things with it". Not all of the good compression algorithms suit seismic imaging. As Yves Meyer pointed out, "An algorithm that is optimal for compression can be disastrous for analysis" (Meyer, 1993). We need to find the best algorithm not only for data compression but also for compression of migration operators. In this work we study one-way wave propagation operators in the wavelet domain, and their compression in different wavelet bases.

Beamlet Decomposition and Compression of Migrators

As an example, we study the one-way wave propagation and imaging using Kirchhoff integral (or Rayleigh integral for flat surfaces):

$$m = \int_s ds \left[\frac{\partial u}{\partial n} g - u \frac{\partial g}{\partial n} \right] = -2 \int_s ds u \frac{\partial g}{\partial n} \quad (1)$$

where m is the extrapolated wave field for forward problem, and the image function for imaging problem, s is the surface on which the wave field u (pressure) is known, n is the normal of the surface, and g is the Green's function. If we replace the Green's function in above formulas by its conjugate (changing forward propagation into backpropagation), the surface integrals become migration operators.

For the sake of simplicity, we use the Kirchhoff migrator in homogeneous background (Rayleigh backpropagation integral) as an example. Figure 1a and 1b show the matrix representations of a full-aperture migrator for different frequencies (5.9 Hz and 25 Hz). This matrix will migrate a wave field 128 points long with sampling interval $\Delta x = 25$ m to a depth of $z = 25$ m. In the figures only real parts of the complex operators are shown. We see that the Kirchhoff operator in space domain is a dense matrix, i.e. the off-diagonal coefficients fall off slowly when moving away from the diagonal. The second and third rows of Figure 1 (c-f) show the matrix representations of the same operator in wavelet domain with different bases, Daubechies 4 (D4) and Coiflet 5 (C5), respectively. These figures demonstrate the different behaviors of migrators in wavelet domain of different wavelet bases. A wavelet-decomposed migrator represents the

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backpropagation of beams of different angles in different scales. Therefore we name it as “BEAMLET MIGRATOR”. We see that for sharp and short bases, such as D4, both the interscale coupling (the coupling between large and small angle (scale) beamlets) and intrascale (the coupling between beamlets in the same scale) are strong. However, the interscale coupling is relatively weak for smooth bases, such as higher-order Daubechies wavelets, Coiflets, and spline wavelets. In Figure 1e and 1f is shown the case of C5. Note that the beamlet matrices are highly sparse matrices compared with the original Kirchhoff matrix. This means that great efficiency can be achieved with beamlet migration. The bottom two rows in Figure 1 (g-j) show the compressed beamlet migrators (D4 and C5) with a cut level of one percent of the maximum coefficient. The compression ratio of the migrator ranges from a few times to a few hundred times, depending on the frequency, step length and the wavelet basis.

Imaging with Compressed Beamlet Migrators

The operator compression in the wavelet domain is very different from more traditional techniques for reducing computations for Kirchhoff migration, such as aperture limitation. Aperture limitation can result in degradation of image quality. On the other hand, the compression in wavelet domain will retain the effective aperture untouched. This is due to the efficient way of representing the migration operator in wavelet domain and the multi-scale nature of the wavelet transform. In order to see the effectiveness and the aperture preserved nature of the beamlet migrator compression, in Figure 2 and 3 we show some examples of beamlet migration for simple objects: point scatterers. Figure 2b is the synthetic seismic section of a point scatterer (an exploding reflector), and Figure 2a shows the migrated image by a full aperture Kirchhoff migration, which is served as a reference image to compare with. Figure 2c and 2d show the migrated image by a compressed D4 beamlet migrator and its residual image with respect to the reference image in figure 2a, respectively. The residual image has been amplified 10 times in strength. Figure 2e and 2f are parallel to Figure 2c and 2d but using a compressed C5 beamlet migrator. We see that the images using the compressed beamlet migrators are almost identical to that using the uncompressed migrator. The averaged compression ratios over frequency is 14:1 for the D4 beamlet migrator, and 17:1 for the C5 beamlet migrator. In comparison, Figure 2g and 2h give the migrated image and the residual image, respectively, for a 33 point aperture Kirchhoff migrator which requires a similar amount of computation time as the compressed D4 beamlet migrator. We see that the image quality is much deteriorated due to the limited aperture problem.

Figure 1: Comparison of migration operator in different domains. The left panel is for $f = 5.9$ Hz. and the right panel, $f = 25$ Hz. Only real parts of the complex operator are plotted. (a)-(b): in space domain; (c)-(d): in D4 wavelet domain; (e)-(f): in C5 wavelet domain; (g)-(h): compressed operator in D4 wavelet domain. (i)-(j) compressed operator in C5 wavelet domain. The cut off level is 0.01 of maximum coefficient.

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Figure 2: Performance comparison of compressed migrators in wavelet domain and migrators in space domain. The synthetic zero-offset seismic section for a point scatterer is shown in (b) with 128 geophones, $\Delta x = 25\text{m}$, $f_{max} = 50\text{ Hz}$, and a Ricker source wavelet. (a): Migrated image by a full aperture Kirchhoff migration; (c), (e) and (g) are the migrated images by the compressed D4 beamlet migrator (compression ratio = 14), compressed C5 beamlet migrator (compression ratio = 17) and a 33 point width Kirchhoff migrator, respectively. On the right, (d), (f) and (h) are the corresponding residual images (amplified by a factor of 10).

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As another example, Figure 3a gives the synthetic sex-offset seismic section of three point scatterers and Figure 3b shows the migrated images by the compressed D4 beamlet migrator. We see the high resolution and good image quality have been retained after the operator compression in wavelet domain. In contrast, the images of the three points migrated by the 33 point Kirchhoff migrator become fuzzy and are not separable due to the interference between the poorly-resolved and distorted individual image fields of these points.

Conclusions

We have shown that the compressed b-let matrix is a highly sparse matrix. The beamlet migrator represents the backpropagation of beams of different angles in different scales. For sharp and short bases, such as D4, both the interscale coupling (the coupling between large and small angle (scale) beamlets) and intrascale coupling (the coupling between beamlets in the same scale) are strong. However, the interscale coupling is relatively weak for smooth bases, such as higher-order Daubechies wavelets, Coiflets, and spline wavelets. The images obtained by the compressed beamlet operator is almost identical to the images by a full-aperture Kirchhoff migration. Compared with the limited aperture Kirchhoff migration, beamlet migration can obtain much wider effective apertures, hence higher resolution and image quality, with similar computational efficiency.

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Figure 3: Synthetic zero-offset seismic section for three point scatterers; (b) Migrated images by the compressed D4 beamlet migrator ($\Delta x = \Delta z = 12.5m$, $f_{max} = 50Hz$), with a compression ratio of 14:1; (c) Migrated images by the 33 point Kirchhoff migrator (same parameters).