ONE-WAY AND ONE-RETURN APPROXIMATIONS (DE WOLF APPROXIMATION) FOR FAST ELASTIC WAVE MODELING IN COMPLEX MEDIA

RU-SHAN WU, XIAO-BI XIE AND XIAN-YUN WU

*Modeling and Imaging Laboratory, Institute of Geophysics and Planetary Physics,
University of California, Santa Cruz, California, USA*

ABSTRACT

The De Wolf approximation has been introduced to overcome the limitation of the Born and Rytov approximations in long range forward propagation and backscattering calculations. The De Wolf approximation is a multiple-forescattering-single-backscattering (MFSB) approximation, which can be implemented by using an iterative marching algorithm with a single backscattering calculation for each marching step (a thin-slab). Therefore, it is also called a one-return approximation. The marching algorithm not only updates the incident field step-by-step, in the forward direction, but also the Green's function when propagating the backscattered waves to the receivers. This distinguishes it from the first order approximation of the asymptotic multiple scattering series, such as the generalized Bremmer series, where the Green's function is approximated by an asymptotic solution. The De Wolf approximation neglects the reverberations (internal multiples) inside thin-slabs, but can model all the forward scattering phenomena, such as focusing/defocusing, diffraction, refraction, interference, as well as the primary reflections.

In this chapter, renormalized MFSB (multiple-forescattering–single-backscattering) equations and the dual-domain expressions for scalar, acoustic and elastic waves are derived by using a unified approach. Two versions of the one-return method (using MFSB approximation) are given: one is the wide-angle, dual-domain formulation (thin-slab approximation) (compared to the screen approximation, no small-angle approximation is made in the derivation); the other is the screen approximation. In the screen approximation, which involves a small-angle approximation for the wave-medium interaction, it can be clearly seen that the forward scattered, or transmitted waves are mainly controlled by velocity perturbations; while the backscattered or reflected waves, are mainly controlled by impedance perturbations. Later in this chapter the validity of the thin-slab and screen methods, and the wide-angle capability of the dual-domain implementation are demonstrated by numerical examples. Reflection coefficients of a plane interface, derived from numerical simulations by the wide-angle method, are shown to match the theoretical curves well up to critical angles. The methods are applied to the fast calculation of synthetic seismograms. The results are compared with finite difference (FD) calculations for the elastic French model. For weak heterogeneities ($\pm 15\%$ perturbation),

good agreement between the two methods verifies the validity of the one-return approach. However, the one-return approach is about 2–3 orders of magnitude faster than the elastic FD algorithm. The other example of application is the modeling of amplitude variation with angle (AVA) responses for a complex reservoir with heterogeneous overburdens. In addition to its fast computation speed, the one return method (thin-slab and complex-screen propagators) has some special advantages when applied to the thin-bed and random layer responses.

Keywords: One-way wave equation, Generalized screen propagator, Seismic wave modeling, AVO

1. INTRODUCTION

One-way approximation for wave propagation has been introduced and widely used as propagators in forward and inverse problems of scalar, acoustic and elastic waves (e.g., Claerbout, 1970, 1976; Landers and Claerbout, 1972; Flatté and Tappert, 1975; Corones, 1975; Tappert, 1977; McCoy, 1977; Hudson, 1980; Ma, 1982; Wales and McCoy, 1983; Fishman and McCoy, 1984, 1985; Wales, 1986; McCoy and Frazer, 1986; Collins, 1989, 1993; Collins and Westwood, 1991; Stoffa *et al.*, 1990; Fisk and McCartor, 1991; Wu and Huang, 1992; Ristow and Ruhl, 1994; Wu, 1994, 1996, 2003; Wu and Xie, 1994; Huang and Wu, 1996; Wu and Jin, 1997; Grimbergen *et al.*, 1998; Van Stralen *et al.*, 1998; Wild and Hudson, 1998; Huang *et al.*, 1999a, 1999b; Thomson, 1999, 2005; De Hoop *et al.*, 2000; Lee *et al.*, 2000; Wild *et al.*, 2000; Wu *et al.*, 2000a, 2000b; Le Rousseau and De Hoop, 2001; Wu and Wu, 2001, 2006; Xie and Wu, 2001, 2005; Han and Wu, 2005). The great advantages of one-way propagation methods are the fast speed of computation, often by several orders of magnitudes faster than the full wave finite difference and finite element methods, and the huge saving in internal memory. The successful extension and applications of one-way elastic wave propagation methods has stimulated the research interest in developing similar theory and techniques for reflected or backscattered wave calculation. There are several approaches in extending the one-way propagation method to include backscattering and multiple scattering calculations. The key difference between these approaches is how to define a reference Green's function for constructing one-way propagators. The generalized Bremmer series (GBS) approach (Corones, 1975; De Hoop, 1996; Wapenaar, 1996, 1998; Van Stralen *et al.*, 1998; Thomson, 1999; Le Rousseau and De Hoop, 2001) adopts an asymptotic solution of the acoustic or elastic wave equation in the heterogeneous medium as the Green's function, i.e. the one-way propagator in the preferred direction. The multiple scattering series is based on the interaction of Green's field (incident field) with the medium heterogeneities. The other approach, i.e. the generalized screen propagator (GSP) approach (Wu, 1994, 1996, 2003; Wu and Xie, 1993, 1994; Wu *et al.*,

1995; Wild and Hudson, 1998; De Hoop *et al.*, 2000; Wild *et al.*, 2000; Xie *et al.*, 2000; Xie and Wu, 2001, 2005), on the other hand, does not use asymptotic solutions. Instead, the approach uses the multiple-forward-scattering (MFS) corrected one-way propagator as the Green's function. When the backscattered field, calculated at each thin-slab, is propagated to the backward direction, the same MFS corrected one-way propagator is used. In surface wave modeling, Friederich *et al.* (1993) and Friederich (1999) adopted a similar approach of MFS approximation (see Chapter 2 by Maupin in this book). However, the approach did not apply the MFS correction to the one-way propagator in the backward direction for the backscattered waves, and therefore did not take the full advantages of the De Wolf approximation. In Section 2, we will compare GBS and GSP approaches after introducing the De Wolf multiple scattering series and the related approximation. In the rest of this chapter we will concentrate on the formulation and applications of the generalized screen approach.

In the generalized screen approach, Wu and Huang (1995) introduced a wide-angle modeling method for backscattered acoustic waves using the multiple-forward-scattering approximation and a phase-screen propagator. Xie and Wu (1995; 2001) extended the complex screen method to include the calculation of backscattered elastic waves under the small-angle approximation. Wu (1996) derived a more general theory for acoustic and elastic waves using the De Wolf approximation, and the theory provided two versions of algorithms: the thin-slab method and the complex-screen method. Later, Wu and Wu (1999, 2003a) introduced a fast implementation of the thin-slab method and a second order improvement for the complex-screen method. In Section 3, the dual-domain thin-slab formulations for the case of scalar, acoustic and elastic media are derived. In Section 4, a fast implementation of the thin-slab propagator is presented with numerical examples. The excellent agreement between the thin-slab and the elastic FD method in the numerical examples demonstrates the validity and efficiency of the theory and method. In Section 5, the small-angle approximation is introduced to derive the screen approximation, which is less accurate for wide angle scattering but is more efficient than the thin-slab method. The validity and potential of the one return approach and the wide-angle capability for the dual-domain implementation are demonstrated by numerical examples for both thin-slab and screen methods applying to the calculation of synthetic seismic sections. In Section 6, the thin-slab method is applied to wave field and amplitude variation with offset (AVO) modeling in exploration seismology.

2. BORN, RYTOV, DE WOLF APPROXIMATIONS AND MULTIPLE SCATTERING SERIES

The perturbation approach is one of the well-known approaches for wave propagation, scattering and imaging (see Chapter 9 of Morse and Feshbach, 1953;

Chapter 13 of Aki and Richards, 1980; Wu, 1989). Traditionally, the perturbation method was used only for weakly inhomogeneous media and short propagation distance. However, recent progress in this direction has led to the development of iterative perturbation solutions in the form of a one-way marching algorithm for scattering and imaging problems in strongly heterogeneous media. For a historical review, see Section 3.1 of Wu (2003). In this section, we give a theoretical analysis of the perturbation approach, including the Born, Rytov and De Wolf approximations, as well as the multiple scattering series. The relatively strong and weak points of the Born and Rytov approximations are analyzed. Since the Born approximation is a weak scattering approximation, it is not suitable for large volume or long-range numerical simulations. The Rytov approximation is a smooth scattering approximation, which works well for long-range small-angle propagation problems, but is not applicable to large-angle scattering and backscattering. Then, the De Wolf approximation (multiple foreshattering single backscattering, or “one-return approximation”) is introduced to overcome the limitations of the Born and Rytov approximations in long range forward propagation and backscattering calculations, which can serve as the theoretical basis of the new dual-domain propagators.

2.1. Born Approximation and Rytov Approximation: Their Strong and Weak Points

For the sake of simplicity, we consider the scalar wave case as an example. The scalar wave equation in inhomogeneous media can be written as

$$\left(\nabla^2 + \frac{\omega^2}{c^2(\vec{r})} \right) u(\vec{r}) = 0, \quad (1)$$

where ω is the circular frequency, $\vec{r}$ is the position vector, and $c(\vec{r})$ is wave velocity at $\vec{r}$. Define c_0 as the background velocity of the medium, resulting in

$$(\nabla^2 + k^2)u(\vec{r}) = -k^2\varepsilon(\vec{r})u(\vec{r}), \quad (2)$$

where $k = \omega/c_0$ is the background wavenumber and

$$\varepsilon(\vec{r}) = \frac{c_0^2}{c^2(\vec{r})} - 1 \quad (3)$$

is the perturbation function (dimensionless force). Set

$$u(\vec{r}) = u^0(\vec{r}) + U(\vec{r}), \quad (4)$$

where $u^0(\vec{r})$ is the unperturbed wave field or “incident wave field” (field in the homogeneous background medium), and $U(\vec{r})$ is the scattered wave field. Substitute (4) into (2) and notice that $u^0(\vec{r})$ satisfies the homogeneous wave equation,

resulting in

$$u(\vec{r}) = u^0(\vec{r}) + k^2 \int_V d^3\vec{r}' g(\vec{r}; \vec{r}') \varepsilon(\vec{r}') u(\vec{r}'), \quad (5)$$

where $g(\vec{r}; \vec{r}')$ is the Green's function in the reference (background) medium, and the integral is over the whole volume of medium. This is the Lippmann–Schwinger integral equation. Since the field $u(\vec{r})$ under the integral is the total field which is unknown, Eq. (5) is not an explicit solution but an integral equation.

2.1.1. Born Approximation

Approximating the total field under the integral with the incident field $u^0(\vec{r})$, we obtain the Born approximation

$$u(\vec{r}) = u^0(\vec{r}) + k^2 \int_V d^3\vec{r}' g(\vec{r}; \vec{r}') \varepsilon(\vec{r}') u^0(\vec{r}'). \quad (6)$$

In general, the Born approximation is a weak scattering approximation, which is only valid when the scattered field is much smaller than the incident field. This implies that the heterogeneities are weak and the propagation distance is short. However, the valid regions of the Born approximation are very different for forward scattering than for backscattering. Forward scattering divergence or catastrophe is the weakest point of Born approximation. For simplicity, we use “forescattering” to stand for “forward scattering”. As can be seen from Eq. (6), the total scattering field is the sum of scattered fields from all parts of the scattering volume. Each contribution is independent from other contributions since the incident field is not updated by the scattering process. In the forward direction, the scattered fields from each part propagate with the same speed as the incident field, so they will be coherently superposed, leading to the linear increase of the total field. The Born approximation does not obey energy conservation. The energy increase will be the fastest in the forward direction, resulting in a catastrophic divergence for long distance propagation. On the contrary, backscattering behaves quite differently from forescattering. Since there is no incident wave in the backward direction, the total observed field is the sum of all the backscattered fields from all the scatterers. However, the size of coherent stacking for backscattered waves is about $\lambda/4$ because of the two-way travel time difference. Beyond this coherent region, all other contributions will be cancelled out. For this reason, backscattering does not have the catastrophic divergence even when the Born approximation is used. This can be further explained with the spectral responses of heterogeneities to scatterings with different scattering angles.

From the analysis of scattering characteristics, we know that the forescattering is controlled by the d.c. component of the medium spectrum $W(0)$, but the backscattering is determined by the spectral component at spatial frequency $2k$, i.e. $W(2k)$, where k is the background wavenumber (see Wu and Aki, 1985;

Wu, 1989). The d.c. component of the medium spectrum is linearly increasing along the propagation distance, while the contribution from $W(2k)$ is usually much smaller and increases much slower than $W(0)$. The validity condition for the Born approximation is the smallness for the scattered field compared with the incident field. Therefore, the region of validity for the Born approximation for backscattering is much larger than that for foreshattering. The other difference between backscattering and foreshattering is their responses to different types of heterogeneities. The backscattering is sensitive to the impedance type of heterogeneities, while foreshattering mainly responds to velocity type of heterogeneities. Velocity perturbation will produce travel-time or phase change, which can accumulate to quite large values, causing the breakdown of the Born approximation. This kind of phase-change accumulation can be easily handled by the Rytov transformation. This is why the Rytov approximation performs better than the Born approximation for foreshattering and has been widely used for long range propagation in the case of foreshattering or small-angle scattering dominance.

2.1.2. Rytov Approximation

Let $u^0(\vec{r})$ be the solution in the absence of perturbations, i.e.,

$$(\nabla^2 + k^2)u^0 = 0, \quad (7)$$

and the perturbed wave field after interaction with the heterogeneity as $u(\vec{r})$. We normalize $u(\vec{r})$ by the unperturbed field $u^0(\vec{r})$ and express the perturbation of the field by a complex phase perturbation function $\psi(\vec{r})$, i.e.,

$$\frac{u(\vec{r})}{u^0(\vec{r})} = e^{\psi(\vec{r})}. \quad (8)$$

This is the Rytov Transformation (see Tatarskii, 1971; or Ishimaru, 1978, p. 349). $\psi(\vec{r})$ denotes the phase and log-amplitude deviations from the incident field:

$$\psi = \log u - \log u^0 = \log \left[\frac{A}{A^0} \right] + i(\phi - \phi^0), \quad (9)$$

where ϕ and ϕ^0 are phases of perturbed and unperturbed waves. Combining (2), (7) and (8) yields

$$2\nabla u^0 \cdot \nabla \psi + u^0 \nabla^2 \psi = -u^0 (\nabla \psi \cdot \nabla \psi + k^2 \epsilon). \quad (10)$$

The simple identity

$$\nabla^2 (u^0 \psi) = \psi \nabla^2 u^0 + 2\nabla u^0 \cdot \nabla \psi + u^0 \nabla^2 \psi,$$

together with (7) results in

$$2\nabla u^0 \cdot \nabla \psi + u^0 \nabla^2 \psi = (\nabla^2 + k^2)(u^0 \psi). \quad (11)$$

From (10) and (11) we obtain

$$(\nabla^2 + k^2)(u^0\psi) = -u^0(\nabla\psi \cdot \nabla\psi + k^2\varepsilon). \quad (12)$$

The solution of (12) can be expressed as an integral equation:

$$u^0(\vec{r})\psi(\vec{r}) = \int_V d^3\vec{r}' g(\vec{r}; \vec{r}') u^0(\vec{r}') [\nabla\psi(\vec{r}') \cdot \nabla\psi(\vec{r}') + k^2\varepsilon(\vec{r}')], \quad (13)$$

where $g(\vec{r}; \vec{r}')$ is the Green's function for the background medium, $u^0(\vec{r}')$ and $u^0(\vec{r})$ are the incident field at $\vec{r}'$ and $\vec{r}$, respectively.

Equation (13) is a nonlinear (Ricatti) equation. Assuming $|\nabla\psi \cdot \nabla\psi|$ is small with respect to $k^2|\varepsilon|$, we can neglect the term $\nabla\psi \cdot \nabla\psi$ and obtain a solution known as the *Rytov approximation*:

$$\psi(\vec{r}) = \frac{k^2}{u^0(\vec{r})} \int_V d^3\vec{r}' g(\vec{r}; \vec{r}') \varepsilon(\vec{r}') u^0(\vec{r}'). \quad (14)$$

Now, we discuss the relationship between the Rytov and Born approximations, and their strong and weak points, respectively. By expanding e^ψ into a power series, the scattered field can be written as

$$u - u^0 = u^0(e^\psi - 1) = u^0\psi + \frac{1}{2}u^0\psi^2 + \dots \quad (15)$$

When $\psi \ll 1$, i.e., the accumulated phase change is less than one radian (corresponding to about one sixth of the wave period), the terms of ψ^2 and higher terms can be neglected, and

$$u - u^0 = u^0\psi = k^2 \int_V d^3\vec{r}' g(\vec{r}; \vec{r}') \varepsilon(\vec{r}') u^0(\vec{r}'), \quad (16)$$

which is the Born approximation. This indicates that when $\psi \ll 1$, the Rytov approximation reduces to the Born approximation. In the case of large phase-change accumulation, the Born approximation is no longer valid. The Rytov approximation still holds as long as the condition $|\nabla\psi \cdot \nabla\psi| \ll k^2|\varepsilon|$ is satisfied.

Let us look at the implication of the condition $|\nabla\psi \cdot \nabla\psi| \ll k^2|\varepsilon|$ for the Rytov approximation. Assume that the observed total field after wave interacted with the heterogeneities is nearly a plane wave:

$$u = Ae^{i\vec{k}_1 \cdot \vec{r}},$$

which could be the refracted wave in the forward direction, or the backscattered field, where $\vec{k}_1 = k\hat{k}_1$ and $\hat{k}_1$ is a unit vector. Since the incident wave is

$$u^0 = A_0 e^{i\vec{k}_0 \cdot \vec{r}},$$

the complex phase field ψ can be written as

$$\psi = \log(A/A_0) + i(\vec{k}_1 - \vec{k}_0) \cdot \vec{r}, \quad (17)$$

and

$$\nabla\psi = \nabla \log(A/A_0) + i(\vec{k}_1 - \vec{k}_0), \quad (18)$$

$$\begin{aligned} \nabla\psi \cdot \nabla\psi &= \left| \nabla \log(A/A_0) \right|^2 - |\vec{k}_1 - \vec{k}_0|^2 \\ &\quad + 2i(\vec{k}_1 - \vec{k}_0) \cdot \nabla \log(A/A_0). \end{aligned} \quad (19)$$

Normally wave amplitudes vary much slower than the phases, so the major contribution to $\nabla\psi \cdot \nabla\psi$ in (19) is from the phase term $|\vec{k}_1 - \vec{k}_0|^2$. Therefore, the condition for Rytov approximation can be approximately stated as

$$|\vec{k}_1 - \vec{k}_0|^2 = 4k^2 \sin^2 \frac{\theta}{2} \ll k^2 |\varepsilon|, \quad (20)$$

where θ is the scattering angle. Therefore the Rytov approximation is only valid when the scattering angle (deflection angle) is small enough to satisfy

$$\sin \frac{\theta}{2} \ll \sqrt{\frac{1}{4}\varepsilon} = \frac{1}{2} \sqrt{\frac{c_0^2 - c^2(\vec{r})}{c^2(\vec{r})}}. \quad (21)$$

This is a point-to-point analysis of the contributions from different terms (for example, the terms in the differential equation (12)). For the integral equation (13), one needs estimate the integral effects of $\nabla\psi \cdot \nabla\psi$ and $k^2\varepsilon$. The heterogeneities need to be smooth enough to guarantee the smallness of the integral of $\nabla\psi \cdot \nabla\psi$ which is related to scattering angles, in comparison with the total scattering contribution $k^2\varepsilon$. In any case, the Rytov approximation is totally inappropriate for backscattering. In the exactly backward direction, $\theta = 180^\circ$ and $\sin \frac{\theta}{2} = 1$, inequality (21) is hardly satisfied. Therefore, although not explicitly specified, the Rytov approximation is a kind of small angle approximation. Together with the parabolic approximation, they formed a set of analytical tools widely used for the forward propagation and scattering problems, such as the line-of-sight propagation problem (e.g., Flatté *et al.*, 1979; Ishimaru, 1978; Tatarskii, 1971). The Rytov approximation is also used in modeling transmission fluctuation for seismic array data (Flatté and Wu, 1988; Wu and Flatté, 1990), diffraction tomography (Devaney, 1982, 1984; Wu and Toksöz, 1987). Tatarskii (1971, Chapter 3B) has some discussions on the relation between the Rytov approximation and the parabolic approximation.

2.2. De Wolf Approximation

We see the limitations of both the Born and Rytov approximations. Even in weakly inhomogeneous media, we need better tools for wave modeling and imaging for long distance propagation. Higher order terms of the Born series (defined later in this section) may help in some cases. However, for strong scattering media, Born series will either converge very slowly, or become divergent. That is

because the Born series is a global interaction series, each term in the series is global in nature. The first term of the Born approximation is a global response, and the higher terms are just global corrections. If the first term has a big error, it will be hard to correct it with higher order terms. One solution to the divergence of the scattering series is the renormalization procedure. Renormalization methods try to split the operations so that the scattering series can be reordered into many sub-series. We hope that some sub-series can be summed up theoretically so that the divergent elements of the series can be removed. The De Wolf approximation splits the scattering potential into foreshattering and backscattering parts and renormalizes the incident field and Green's function into the forward propagated field and forward propagated Green's function (forward propagator), respectively (De Wolf, 1971, 1985). The forward propagated field u_f is the sum of an infinite sub-series including all the multiple foreshattered fields. The forward propagator G_f is the sum of a similar sub-series including multiple foreshattering corrections to the Green's function. For backscattering, only the single backscattered field is calculated at each step, and then propagated in the backward direction using the renormalized forward propagator (Green's function) G_f . The *De Wolf approximation* is also called the “*one-return approximation*” (Wu, 1996, 2003; Wu and Huang, 1995; Wu *et al.*, 2000a, 2000b), since it is a *multiple-foreshattering–single-backscattering (MFSB) approximation*. It is also a kind of *local* Born approximation since the Born approximation applies only locally to the individual thin-slabs. From previous sections we know that the Born approximation works well for backscattering. With the renormalized incident field and Green's function, the local Born (MFSB) proved to work surprisingly well for many practical applications. The key is to have good forward propagators. Rino (1988) has obtained better approximation than MFSB in the wavenumber domain and pointed out the error of the De Wolf approximation in the calculation of backscattering enhancement. The error (overestimate) is again due to the violation of the energy conservation law by the Born approximation. Even with a foreshattering correction, the backscattered energy is still not removed from the forward propagated waves for local Born approximation. However, for short propagation distances in exploration seismology and some other applications, the errors in reflection amplitudes may not become a serious problem.

In the following, we will adopt an intuitive approach of derivation to see the physical meaning of the approximation. The De Wolf approximation bears some similarity to the Twersky approximation in the case of discrete scatterers (Twersky, 1964; Ishimaru, 1978). The Twersky approximation includes all the multiple scattering, except the reverberations between pairs of scatterers that excludes the paths which connect the two neighboring scatterers more than once. The Twersky approximation has less restrictions and therefore a wider range of applications than the De Wolf approximation. The latter needs define the split of forward and

back scatterings. We define the scattering to the forward hemisphere as foreshattering and its complement as backscattering.

The Lippmann–Schwinger equation (5) can be written symbolically as

$$u = u^0 + G_0 \varepsilon u, \quad (22)$$

where ε is a diagonal operator in space domain, and G_0 is a nondiagonal integral operator. If the reference medium is homogeneous, G_0 will be the volume integral with the Green's function $g_0(\vec{r}; \vec{r}')$ as the kernel. Formally, (22) can be expanded into infinite scattering series (Born series)

$$u = u^0 + G_0 \varepsilon u^0 + G_0 \varepsilon G_0 \varepsilon u^0 + \cdots. \quad (23)$$

If we split the scattering potential into the foreshattering and backscattering parts

$$\varepsilon = \varepsilon_f + \varepsilon_b \quad (24)$$

and substitute it into (23), we can have all combinations of multiple foreshattering and backscattering. We neglect the multiple backscattering (reverberations), i.e., drop all the terms containing two or more backscattering potentials, resulting in a multiple scattering series which contains terms with only one ε_b .

The general term will look like

$$G_0 \varepsilon_f G_0 \varepsilon_f \cdots G_0 \varepsilon_b G_0 \varepsilon_f \cdots G_0 \varepsilon_f u^0. \quad (25)$$

The multiple foreshattering on the left side of ε_b can be written as

$$G_f^m = [G_0 \varepsilon_f]^m G_0, \quad (26)$$

and on its right side,

$$u_f^n = [G_0 \varepsilon_f]^n u^0. \quad (27)$$

Collecting all the terms of G_f^m and u_f^n , respectively, we have

$$\begin{aligned} G_f^M &= \sum_{m=0}^M [G_0 \varepsilon_f]^m G_0, \\ u_f^N &= \sum_{n=0}^N [G_0 \varepsilon_f]^n u^0. \end{aligned} \quad (28)$$

Let M and N go to infinite, then the renormalized G_f (forward propagator) and u_f (foreshattering corrected incident field) are:

$$G_f = \sum_{m=0}^{\infty} [G_0 \varepsilon_f]^m G_0,$$

$$u_f = \sum_{n=0}^{\infty} [G_0 \varepsilon_f]^n u^0, \quad (29)$$

and the De Wolf approximation (in operator form) becomes

$$u = u_f + G_f \varepsilon_b u_f. \quad (30)$$

The observed total field u in (30) is different for different observation geometries. For transmission problems, the backscattering potential does not have any effect under the De Wolf approximation,

$$u_{\text{transmission}} = u_f. \quad (31)$$

On the other hand, for reflection measurement, that is, when the observations are in the same level as or behind the source with respect to the propagation direction, there is no u_f in the total field,

$$u_{\text{reflection}} = G_f \varepsilon_b u_f. \quad (32)$$

Write it into integral form, (30) becomes

$$u(\vec{r}) = u_f(\vec{r}) + \int_V d^3 \vec{r}' g_f(\vec{r}, \vec{r}') \varepsilon_b(\vec{r}') u_f(\vec{r}'). \quad (33)$$

Note that both the incident field and the Green's function have been renormalized by the multiple foreshattering process through the multiple interactions with the forward scattering potential ε_f .

2.3. The De Wolf Series (DWS) of Multiple Scattering

De Wolf approximation can be considered as the first term of a multiple scattering series: the De Wolf series. After substituting the decomposition (24) into the Born series (23), we rearrange and recompose the scattering series into a series according to the power of the backscattering potential ε_b . The first order in ε_b will be the De Wolf approximation (one-return approximation). The second order corresponds to the double backscattering (double reflection, double return) term. The higher terms represent the multiple backscattering series. The whole multiple scattering series (23) can be reorganized into

$$\begin{aligned} u &= u_f + G_f \varepsilon_b u_f + G_f \varepsilon_b G_f \varepsilon_b u_f + \cdots \\ &= \sum_{m=0}^M [G_f \varepsilon_b]^m u_f. \end{aligned} \quad (34)$$

We see that the zero order term is the forward scattering approximated direct wavefield. It is the direct transmitted wave in the real media. The first order term is the De Wolf approximation, which corresponds to the single backscattering

signal, or the primary reflections, as called in exploration seismology. This single backscattering signal is different from the Born approximation where the incident field and the Green's function are both defined in the background medium (a homogeneous medium).

The De Wolf Series and Generalized Bremmer Series

Here we point out the differences between the De Wolf series (DWS) and the Generalized Bremmer Series (GBS). The original Bremmer series (Bremmer, 1951) is a geometric-optical series for stratified media, which can be considered as a higher order extension to the regular WKBJ solution (the first order term). Later it was generalized to 3-D inhomogeneous media, and was named the generalized Bremmer series (Corones, 1975; De Hoop, 1996; Wapenaar, 1996, 1998; Van Stralen *et al.*, 1998; Thomson, 1999; Le Rousseau and De Hoop, 2001). The zero order term (the leading term) of the GBS is a high-frequency asymptotic solution (a WKBJ-like solution or Rytov-like solution) (De Hoop, 1996), and used as the Green's function for deriving the higher order terms. The Green's function is not updated when calculating the higher order scattering. Therefore, it is similar to the Born series in a certain sense. Unlike the DWS, which is a series in terms of medium velocity variation, the GBS is in terms of the spatial derivatives of the medium properties (De Hoop, 1996). Because of the asymptotic nature of its Green's function, the media need to be "smooth" on the scale of the irradiating pulse (De Hoop, 1996; Van Stralen *et al.*, 1998; Thomson, 1999). Some authors used an equivalent medium averaging process to smooth the medium before the application of the method (Van Stralen *et al.*, 1998). Wapenaar's approach (1996, 1998) is to get an asymptotic solution, without averaging the medium, using the flux normalized decomposition of wavefield. Thomson (1999, 2005) included the second term of the asymptotic series to the asymptotic Green's function to improve the amplitude accuracy. This zero-order term solution for generally inhomogeneous media can be useful for seismic imaging and inversion (e.g. Berkhout, 1982; Berkhout and Wapenaar, 1989). With careful amplitude correction, this type of Green's function is an energy-flux conserved propagator in any heterogeneous media (including discontinuities), and is called (Wu *et al.*, 2004; Wu and Cao, 2005) the "transparent propagator". The first term in the GBS (De Hoop, 1996; Wapenaar, 1996) in fact is not the primary reflection from the real media, but a "primary reflection" on the basis of the asymptotic Green's function. Because the incident field and Green's function are not updated in deriving the first order term, the "primary reflection" thus obtained is similar to a "distorted primary reflection", following the term of "distorted Born approximation" in the physics literature. The real primary reflection, which is the first term in the De Wolf series, includes some multiple scattering terms in the GBS.

3. A DUAL-DOMAIN THIN-SLAB FORMULATION FOR ONE-RETURN (MFSB) SYNTHETICS

The physical meaning of the one-return approximation is shown in Fig. 1. First a preferred direction needs to be selected for the forward/backward scattering decomposition. In Fig. 1 we choose the z -direction as the preferred one. We see that all the solid wave paths have only one backscattering with respect to the z -direction, and therefore will be included in the simulation using the one-return approximation. On the other hand, the dashed line has three backscattering points and cannot be modeled by the one-return approximation. Figure 2 illustrates schematically the numerical implementation of one-return approximation using a thin-slab marching algorithm. The heterogeneous medium is sliced into numerous thin-slabs. Within each thin-slab, the local Born approximation can be utilized for the calculation of the forward and backward scatterings. The foreshattered field is used to update the incident field (the forward propagated wavefield u_f) and the backscattered field u_b is stored for later use. This procedure is iteratively done slab by slab, until the end of the model is reached. At the bottom of the model, u_f will be the primary transmitted wave. To get the primary reflected waves, the marching will be done in the reversed direction, i.e. from the bottom to the top of the model. At each thin-slab, the stored backscattered wavefield will be picked up and propagated to the receiving point with the foreshattering updated Green's function G_f . In the next few sections, the derivation and formulations of the thin-slab and screen approximations for scalar, acoustic and elastic waves will be given.

FIG. 1. Sketch showing the meaning of the De Wolf (one-return) approximation.

3.1. The Case of Scalar Media

Based on the De Wolf approximation (33) and the marching algorithm shown in Fig. 2, at each step we need calculate the fore- and backscattered wave fields caused by the thin-slab. We can choose the thickness of the thin-slab to be thin enough so that the local Born approximation can be applied to the calculation. Replacing u^0 in the Born approximation (6) with the updated local incident field u^f , the scalar pressure field at an observation point $\mathbf{x}^*$ can be expressed as

$$p(\mathbf{x}^*) = p^f(\mathbf{x}^*) + k^2 \int_V d^3\mathbf{x} g(\mathbf{x}^*; \mathbf{x}) F(\mathbf{x}) p^f(\mathbf{x}), \quad (35)$$

where p^f is the local incident pressure field and g , the Green's function in the thin-slab. $F(\mathbf{x})$ is the perturbation function (scattering potential),

$$F(\mathbf{x}) = \frac{c_0^2}{c^2(\mathbf{x})} - 1 = \frac{s^2(\mathbf{x}) - s_0^2}{s_0^2}, \quad (36)$$

with $s = 1/c$ as the slowness, where c is the velocity. The second term in the right-hand side of (35) is the scattered field and the volume integration is over the thin-

FIG. 2. Schematic illustration of the thin-slab method for implementing the one-return approximation: (a) original medium is sliced into thin-slabs, (b) iterative procedure of transmitted and reflected wave calculations by the one-return approximation.

slab. Choosing the z -direction as the main propagation direction, the scattered field at a receiving point $(\mathbf{x}_T^*, z^*)$ can be calculated as

$$P(\mathbf{x}_T^*, z^*) = k^2 \int_V d^3\mathbf{x} g(\mathbf{x}_T^*, z^*; \mathbf{x}) F(\mathbf{x}) p^f(\mathbf{x}), \quad (37)$$

where $\mathbf{x}_T^*$ is the horizontal position in the receiver plane at depth z^* . In the derivations of the next few sections, we use a thin-slab geometry as illustrated in Fig. 3. Within the slab the Green's function is assumed to be a constant medium Green's function g^0 . Set z' and z_1 as the slab entrance (top) and exit (bottom), respectively, and Fourier-transform equation (37) with respect to $\mathbf{x}_T^*$, resulting in

$$P(\mathbf{K}_T, z^*) = k^2 \int_{z'}^{z_1} dz \iint d^2\mathbf{x}_T g^0(\mathbf{K}_T, z^*; \mathbf{x}) F(\mathbf{x}) p^f(\mathbf{x}), \quad (38)$$

where

$$g^0(\mathbf{K}_T, z^*; \mathbf{x}_T, z) = \frac{i}{2\gamma} e^{i\gamma|z^*-z|} e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \quad (39)$$

is the wavenumber-domain Green's function in constant media (see Berkhout, 1987; Wu, 1994), and

$$\gamma = \sqrt{k^2 - \mathbf{K}_T^2} \quad (40)$$

is the vertical wavenumber (or the propagating wavenumber). Substituting (39) into (38) yields

FIG. 3. Geometry for the derivation of the thin-slab method.

$$P(\mathbf{K}_T, z^*) = \frac{i}{2\gamma} k^2 \int_{z'}^{z_1} dz e^{i\gamma|z^*-z|} \iint d^2x_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \times [F(\mathbf{x}_T, z) p^f(\mathbf{x}_T, z)]. \quad (41)$$

Note that the two dimensional inner integral is a 2-D Fourier transform. Therefore, the dual-domain technique can be used to implement (41). If $\mathbf{x}^* = \mathbf{x}_1$, (41) is used to calculate the forward scattered field and update the incident field (transmitted waves) (35); on the other hand, if $\mathbf{x}^* = \mathbf{x}'$, (41) is for the calculation of backscattered waves. In the case that only forward propagation is concerned, the iterative implementation of (35) and (41) composes a *one-way propagator*. The derivation of (35) and (41) is based on the local Born approximation, however, the implementation in dual domains is similar to the classical phase-screen approach of a one-way propagator. It is known that the Born approximation is basically a low-frequency approximation, and has severe phase errors for strong contrast and high-frequencies. In order to have better phase accuracy, which is important for imaging (migration), certain high-frequency asymptotic phase-matching has been applied to the local Born solution such that the travel time of the solution match exactly the geometric-optical (ray) travel time in the forward direction. Even a zero-order matching leads to a solution better than the classic phase-screen method, i.e. the spit-step Fourier method (e.g. Stoffa *et al.*, 1990). The method was originally called the “pseudo-screen” method (Wu and De Hoop, 1996; Huang and Wu, 1996; Jin *et al.*, 1998, 1999) to distinguish the new form of screen propagator from the classic phase-screen propagator. The phase-screen has operations only in the space domain so that the phase correction is accurate only for small-angle waves; while the pseudo-screen has operations in both the space and wavenumber domains to improve the accuracy for large-angle waves. The asymptotic phase-matching method used by Jin and Wu (1999) and Jin *et al.* (1998, 1999, 2002) in the hybrid pseudo-screen propagator applies a wavenumber filter in the form of continued fraction expansion and can improve the large-angle wave response significantly. In the method, the wide-angle correction is implemented with an implicit finite difference scheme and the expansion coefficients are optimized by phase-matching. The pseudo-screen propagator belongs to a more general category of generalized screen propagators (GSP) (Wu, 1994, 1996; Xie and Wu, 1998; De Hoop *et al.*, 2000; Le Rousseau and De Hoop, 2001). The hybrid GSP with finite difference wide-angle correction is similar to the Fourier-finite difference (FFD) method (Ristow and Ruhl, 1994).

3.2. The Case of Acoustic Media

For a linear isotropic acoustic medium, the wave equation in frequency domain is

$$\nabla \cdot \frac{1}{\rho} \nabla p + \frac{\omega^2}{\kappa} p = 0, \quad (42)$$

where p is the pressure field, ρ and κ are the density and bulk module of the medium, respectively. Assuming ρ_0 and κ_0 as the parameters of the background medium, equation (42) can be written as

$$\frac{1}{\rho_0} \nabla^2 p + \frac{\omega^2}{\kappa_0} p = - \left[\omega^2 \left(\frac{1}{\kappa} - \frac{1}{\kappa_0} \right) p + \nabla \cdot \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right) \nabla p \right], \quad (43)$$

or

$$(\nabla^2 + k^2) p(\mathbf{x}) = -k^2 F(\mathbf{x}) p(\mathbf{x}), \quad (44)$$

which is the same as the case of scalar media except

$$F(\mathbf{x}) = \delta_\kappa(\mathbf{x}) + \frac{1}{k^2} \nabla \cdot \delta_\rho \nabla, \quad (45)$$

where $F(\mathbf{x})$ is an operator instead of a scalar function, with

$$\delta_\kappa(\mathbf{x}) = \frac{\kappa_0}{\kappa(\mathbf{x})} - 1, \quad (46)$$

and

$$\delta_\rho(\mathbf{x}) = \frac{\rho_0}{\rho(\mathbf{x})} - 1. \quad (47)$$

If ρ is kept constant ($\rho = \rho_0$), then $\delta_\kappa = c_0^2/c^2 - 1$, going back to the scalar medium case. Following derivation of Eq. (41), using the thin-slab geometry and the dual-domain expression for the acoustic media, the scattered pressure field at the receiving depth z^* can be written explicitly as

$$\begin{aligned} P(\mathbf{K}_T, z^*) &= \frac{i}{2\gamma} k^2 \int_{z'}^{z_1} dz e^{i\gamma|z^*-z|} \\ &\times \left\{ \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} [\delta_\kappa(\mathbf{x}_T, z) p^f(\mathbf{x}_T, z)] \right. \\ &\left. + \frac{i}{k} \hat{\mathbf{k}} \cdot \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} [\delta_\rho(\mathbf{x}_T, z) \nabla p^f(\mathbf{x}_T, z)] \right\}, \quad (48) \end{aligned}$$

where

$$\hat{k} = \frac{1}{k}(\mathbf{K}_T, k_z), \quad (49)$$

and $k_z = \pm\gamma$ for forescattering and backscattering, respectively. The incident field $p^f(\mathbf{x}_T, z)$ and its gradient $\nabla p^f(\mathbf{x}_T, z)$ at depth z can be calculated from the field $p^0(\mathbf{x}'_T, z')$ at the slab entrance

$$p^f(\mathbf{x}) = p^f(\mathbf{x}_T, z) = \frac{1}{4\pi^2} \iint d^2\mathbf{K}'_T p^0(\mathbf{K}'_T) e^{i\gamma'(z-z')} e^{i\mathbf{K}'_T \cdot \mathbf{x}_T}, \quad (50)$$

and

$$\nabla p^f(\mathbf{x}) = \frac{ik}{4\pi^2} \iint d^2\mathbf{K}'_T \hat{k}' p^0(\mathbf{K}'_T) e^{i\gamma'(z-z')} e^{i\mathbf{K}'_T \cdot \mathbf{x}_T}, \quad (51)$$

where

$$\hat{k}' = \frac{1}{k}(\mathbf{K}'_T, \gamma'). \quad (52)$$

Numerical Tests for Reflected Waves in Acoustic Media

In the dual-domain thin-slab formulation, no small-angle approximation is made. The only approximation is the smallness of perturbations within each thin-slab so that the background Green's function can be applied and the incident waves can be treated as propagated in the background media. In the limiting case, the thickness of the thin-slab can be shrunk to a one grid step. In that case, the only approximation involved is the dual-domain implementation (or split step algorithm). Numerical tests on the wide-angle version (thin-slab approximation) of the acoustic one-return method showed that the reflection coefficients calculated from the synthetic acoustic records agree well with the theoretical predictions when the incidence angles are smaller than the critical angle in the case of high-velocity layer reflection, and to approximately 70° in the case of low-velocity layer reflection (Wu and Huang, 1992, 1995; Wu, 1996).

3.3. The Case of Elastic Media

The equation of motion in a linear, heterogeneous elastic medium can be written as (Aki and Richards, 1980)

$$-\omega^2 \rho(\mathbf{x}) \mathbf{u}(\mathbf{x}) = \nabla \cdot \sigma(\mathbf{x}), \quad (53)$$

where $\mathbf{u}$ is the displacement vector, σ is the stress tensor (dyadic) and ρ is the density of the medium. Here we assume no body force exists in the medium. We know the stress-displacement relation

$$\sigma(\mathbf{x}) = \mathbf{c}(\mathbf{x}) : \varepsilon(\mathbf{x}) = \frac{1}{2} \mathbf{c} : (\nabla \mathbf{u} + \mathbf{u} \nabla), \quad (54)$$

where $\mathbf{c}$ is the elastic constant tensor of the medium, ε is the strain field, $\mathbf{u} \nabla$ stands for the transpose of $\nabla \mathbf{u}$, and “:” stands for double scalar product of tensors defined through $(\mathbf{ab}) : (\mathbf{cd}) = (\mathbf{b} \cdot \mathbf{c})(\mathbf{a} \cdot \mathbf{d})$. Equation (53) can be written as a wave equation of the displacement field:

$$-\omega^2 \rho(\mathbf{x}) \mathbf{u}(\mathbf{x}) = \nabla \cdot \left[\frac{1}{2} \mathbf{c} : (\nabla \mathbf{u} + \mathbf{u} \nabla) \right]. \quad (55)$$

If the parameters of the elastic medium and the total wave field can be decomposed as

$$\begin{aligned}\rho(\mathbf{x}) &= \rho_0 + \delta\rho(\mathbf{x}), & \mathbf{c}(\mathbf{x}) &= \mathbf{c}_0 + \delta\mathbf{c}(\mathbf{x}), \\ \mathbf{u}(\mathbf{x}) &= \mathbf{u}^0(\mathbf{x}) + \mathbf{U}(\mathbf{x}),\end{aligned}\tag{56}$$

where ρ_0 and $\mathbf{c}_0$ are the parameters of the background medium, $\delta\rho$ and $\delta\mathbf{c}$ are the corresponding perturbations, $\mathbf{u}^0$ is the incident field and $\mathbf{U}$ is the scattered field, then (55) can be rewritten as:

$$-\omega^2\rho_0\mathbf{U} - \nabla \cdot \left[\frac{1}{2}\mathbf{c}_0 : (\nabla\mathbf{U} + \mathbf{U}\nabla) \right] = \mathbf{F},\tag{57}$$

where

$$\mathbf{F} = \omega^2\delta\rho\mathbf{u} + \nabla \cdot [\delta\mathbf{c} : \varepsilon]$$

is the equivalent body force due to scattering.

Similar to (35), we can write the equation of the De Wolf approximation for elastic displacement field as:

$$\begin{aligned}\mathbf{u}(\mathbf{x}_T^*, z^*) &= \mathbf{u}^f(\mathbf{x}_T^*, z^*) + \int_V d^3\mathbf{x} \{ \delta\rho\omega^2\mathbf{u}^f(\mathbf{x}_T, z) \\ &\quad + \nabla \cdot [\delta\mathbf{c} : \varepsilon^f(\mathbf{x}_T, z)] \} \cdot \mathbf{G}^f(\mathbf{x}_T^*, z^*; \mathbf{x}_T, z).\end{aligned}\tag{58}$$

Following the derivation of Eq. (38), we can express the scattered displacement field for a thin-slab in the horizontal wavenumber domain using local Born approximation as:

$$\begin{aligned}\mathbf{U}(\mathbf{K}_T, z^*) &= \int_{z'}^{z_1} dz \iint d^2\mathbf{x}_T \{ \delta\rho\omega^2\mathbf{u}^f(\mathbf{x}_T, z) \\ &\quad + \nabla \cdot [\delta\mathbf{c} : \varepsilon^f(\mathbf{x}_T, z)] \} \cdot \mathbf{G}^0(\mathbf{K}_T, z^*; \mathbf{x}_T, z),\end{aligned}\tag{59}$$

where

$$\begin{aligned}\mathbf{G}^0(z^*, \mathbf{K}_T; z, \mathbf{x}_T) &= \frac{ik_\alpha^2}{2\rho_0\omega^2} \hat{k}_\alpha \hat{k}_\alpha \frac{1}{\gamma_\alpha} e^{i\mathbf{k}_\alpha \cdot \mathbf{r}} + \frac{ik_\beta^2}{2\rho_0\omega^2} (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \frac{1}{\gamma_\beta} e^{i\mathbf{k}_\beta \cdot \mathbf{r}},\end{aligned}\tag{60}$$

where $\mathbf{I}$ is the unit dyadic, and

$$\begin{aligned}\gamma_\alpha &= \sqrt{k_\alpha^2 - K_T^2}, \\ \gamma_\beta &= \sqrt{k_\beta^2 - K_T^2},\end{aligned}\tag{61}$$

where $k_\alpha = \omega/\alpha_0$ and $k_\beta = \omega/\beta_0$ are P and S wavenumbers with α_0 and β_0 as the P and S wave background velocities of the thin-slab, respectively. For isotropic

media,

$$\delta \mathbf{c}(\mathbf{x}) : \varepsilon(\mathbf{x}) = \delta \lambda(\mathbf{x}) |\varepsilon| \mathbf{I} + 2\delta \mu(\mathbf{x}) \varepsilon(\mathbf{x}). \quad (62)$$

Substituting (60) into (59), we can derive the dual-domain expressions for scattered displacement fields in isotropic elastic media.

For P to P scattering:

$$\begin{aligned} & \mathbf{U}^{PP}(K_T, z^*) \\ &= \frac{ik_\alpha^2}{2\gamma_\alpha} \int_{z'}^{z_1} dz e^{ik_z^\alpha(z^*-z)} \left\{ \hat{k}_\alpha \hat{k}_\alpha \cdot \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{\delta \rho(\mathbf{x}_T, z)}{\rho} \mathbf{u}_\alpha^f(\mathbf{x}_T, z) \right. \\ & \quad - \hat{k}_\alpha \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{\delta \lambda(\mathbf{x}_T, z)}{\lambda + 2\mu} \frac{1}{ik_\alpha} \nabla \cdot \mathbf{u}_\alpha^f(\mathbf{x}_T, z) \\ & \quad \left. - \hat{k}_\alpha (\hat{k}_\alpha \hat{k}_\alpha) : \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{\delta \mu(\mathbf{x}_T, z)}{\lambda + 2\mu} \frac{1}{ik_\alpha} \varepsilon_\alpha^f(\mathbf{x}_T, z) \right\}, \quad (63) \end{aligned}$$

with $k_z^\alpha = +\gamma_\alpha$ for forescattering and $k_z^\alpha = -\gamma_\alpha$ for backscattering, and $\hat{k}_\alpha = (\mathbf{K}_T, k_z^\alpha)/k_\alpha$. Note that we replaced ρ_0, λ_0, μ_0 in denominators by $\rho = \rho_0 + \delta\rho, \lambda = \lambda_0 + \delta\lambda$ and $\mu = \mu_0 + \delta\mu$. This replacement is the result of asymptotic matching between the Born approximation for large-angle scattering and the $h - f$ asymptotic travel-time (phase) for forward propagation. It is proved (Wu and Wu, 2003a, 2003b) that with this replacement (asymptotic matching), the phase-shift in the exact forward direction is accurate and the phase error for small angles is reduced compared with the Born approximation. In the meanwhile, the phase error for large angle scattering is much smaller than that of the phase-screen approximation.

In (63) $\mathbf{u}_\alpha^f(\mathbf{x}_T, z)$, $\nabla \cdot \mathbf{u}_\alpha^f(\mathbf{x}_T, z)$ and $\varepsilon_\alpha^f(\mathbf{x}_T, z)$ can be calculated by:

$$\begin{aligned} \mathbf{u}_\alpha^f(\mathbf{x}_T, z) &= \frac{1}{4\pi} \iint d^2 \mathbf{K}'_T e^{i\mathbf{K}'_T \cdot \mathbf{x}_T} u_\alpha^0(\mathbf{K}'_T) e^{i\gamma'_\alpha(z-z')}, \\ \frac{1}{ik_\alpha} \nabla \cdot \mathbf{u}_\alpha^f(\mathbf{x}_T, z) &= \frac{1}{4\pi} \iint d^2 \mathbf{K}'_T e^{i\mathbf{K}'_T \cdot \mathbf{x}_T} \hat{k}'_\alpha \cdot \mathbf{u}_\alpha^0(\mathbf{K}'_T) e^{i\gamma'_\alpha(z-z')}, \\ \frac{1}{ik_\alpha} \varepsilon_\alpha^f(\mathbf{x}_T, z) &= \frac{1}{4\pi} \iint d^2 \mathbf{K}'_T e^{i\mathbf{K}'_T \cdot \mathbf{x}_T} \frac{1}{2} [\hat{k}'_\alpha \mathbf{u}_\alpha^0(\mathbf{K}'_T) + \hat{k}'_\alpha \mathbf{u}_\alpha^0(\mathbf{K}'_T)] e^{i\gamma'_\alpha(z-z')} \\ &= \frac{1}{4\pi} \iint d^2 \mathbf{K}'_T e^{i\mathbf{K}'_T \cdot \mathbf{x}_T} \hat{k}'_\alpha \hat{k}'_\alpha u_\alpha^0(\mathbf{K}'_T) e^{i\gamma'_\alpha(z-z')}, \quad (64) \end{aligned}$$

where $u_\alpha^0(\mathbf{K}'_T) = |\mathbf{u}_\alpha^0(\mathbf{K}'_T)|$ and $\hat{k}'_\alpha = (\mathbf{K}'_T, \gamma'_\alpha)/k_\alpha$.

For P to S scattering:

$$\begin{aligned}
 U^{PS}(\mathbf{K}_T, z^*) &= \frac{ik_\beta^2}{2\gamma_\beta} \int_{z'}^{z_1} dz e^{ik_z^\beta(z^*-z)} \\
 &\times \left\{ (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \cdot \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{\delta\rho(\mathbf{x}_T, z)}{\rho} \mathbf{u}_\alpha^f(\mathbf{x}_T, z) \right. \\
 &\left. - (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \cdot \left[\hat{k}_\beta \cdot \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} 2 \frac{\delta\mu(\mathbf{x}_T, z)}{\mu} \frac{1}{ik_\beta} \varepsilon_\alpha^f(\mathbf{x}_T, z) \right] \right\}, \quad (65)
 \end{aligned}$$

where $\hat{k}_\beta = (\mathbf{K}_T, k_z^\beta)/k_\beta$.

For S to P scattering:

$$\begin{aligned}
 U^{SP}(\mathbf{K}_T, z^*) &= \frac{ik_\alpha^2}{2\gamma_\alpha} \int_{z'}^{z_1} dz e^{ik_z^\alpha(z^*-z)} \left\{ \hat{k}_\alpha \hat{k}_\alpha \cdot \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{\delta\rho(\mathbf{x}_T, z)}{\rho} \mathbf{u}_\beta^f(\mathbf{x}_T, z) \right. \\
 &\left. - \left(\frac{k_\alpha}{k_\beta} \right) \hat{k}_\alpha (\hat{k}_\alpha \hat{k}_\alpha) : \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} 2 \frac{\delta\mu(\mathbf{x}_T, z)}{\mu} \frac{1}{ik_\beta} \varepsilon_\beta^f(\mathbf{x}_T, z) \right\}. \quad (66)
 \end{aligned}$$

For S to S scattering:

$$\begin{aligned}
 U^{SS}(\mathbf{K}_T, z^*) &= \frac{ik_\beta^2}{2\gamma_\beta} \int_{z'}^{z_1} dz e^{ik_z^\beta(z^*-z)} \\
 &\times \left\{ (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \cdot \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{\delta\rho(\mathbf{x}_T, z)}{\rho} \mathbf{u}_\beta^f(\mathbf{x}_T, z) \right. \\
 &\left. - (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \cdot \left[\hat{k}_\beta \cdot \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} 2 \frac{\delta\mu(\mathbf{x}_T, z)}{\mu} \frac{1}{ik_\beta} \varepsilon_\beta^f(\mathbf{x}_T, z) \right] \right\}. \quad (67)
 \end{aligned}$$

In Eqs. (66) and (67), $\mathbf{u}_\beta^f(\mathbf{x}_T, z)$ and $\varepsilon_\beta^f(\mathbf{x}_T, z)$ can be calculated by

$$\begin{aligned}
 \mathbf{u}_\beta^f(\mathbf{x}_T, z) &= \frac{1}{4\pi^2} \iint d^2\mathbf{K}'_T e^{i\mathbf{K}'_T \cdot \mathbf{x}_T} \mathbf{u}_\beta^0(\mathbf{K}'_T) e^{i\gamma'_\beta(z-z')}, \\
 \frac{1}{ik_\beta} \varepsilon_\beta^f(\mathbf{x}_T, z) &= \frac{1}{4\pi^2} \iint d^2\mathbf{K}'_T e^{i\mathbf{K}'_T \cdot \mathbf{x}_T} \frac{1}{2} [\hat{k}'_\beta \mathbf{u}_\beta^0(\mathbf{K}'_T) + \mathbf{u}_\beta^0(\mathbf{K}'_T) \hat{k}'_\beta] e^{i\gamma'_\beta(z-z')}, \quad (68)
 \end{aligned}$$

where $\hat{k}'_\beta = (\mathbf{K}'_T, \gamma'_\beta)/k_\beta$.

3.4. Implementation Procedure of the One-Return Simulation

Under the MFSB approximation we can update the total field with a marching algorithm in the forward direction. We can slice the whole medium into thin-slabs perpendicular to the propagation direction. Weak scattering condition holds for each thin-slab. For each step forward, the forward and backward scattered fields by a thin-slab between z' and z_1 are calculated. The foreshattered field is added to the incident field so that the updated field becomes the incident field for the next thin-slab. The procedure for acoustic and elastic media can be summarized as follows (see Fig. 2). The simplification for the case of scalar media is straightforward.

1. Fourier transform (FT) the incident fields into wavenumber domain at the entrance of each thin-slab.
2. Free propagate in wavenumber domain and calculate the primary fields and its gradients (including strain fields for the case of elastic media) within the slab.
3. Inverse FT these primary fields and its gradients into space domain, and then interact with the medium perturbations: calculation of the distorted fields.
4. FT the distorted fields into wavenumber domain and perform the divergence (and curl, in the case of elastic media) operations to get the backscattered fields. Sum up the scattered fields by all perturbation parameters and multiply it with a weighting factor $i/2\gamma$, then propagate back to the entrance of the slab.
5. Calculate the foreshattered field at the slab exit and add to the primary field to form the total field as the incident field at the entrance of the next thin-slab.
6. Continue the procedure iteratively until the bottom of the model is reached.
7. Propagate the backscattered waves from the bottom up to the surface and sum up the contributions of all the thin-slabs during the propagation.

Note that medium-wave interaction, for the case of acoustic waves, involves vector operations and needs 3 pairs of fast Fourier transforms (FFTs) for each step, while for the case of elastic waves, tensor (strain fields) operations are involved. Due to the symmetric properties of the strain tensors, there are only 6 independent components for each tensor. From (63)–(68) we see that many pairs of FFTs are required for each step and therefore the computation for elastic wave scattering is rather intensive.

4. FAST ALGORITHM OF THE ELASTIC THIN-SLAB PROPAGATOR AND SOME PRACTICAL ISSUES

4.1. Fast Implementation in Dual Domains

From Eqs. (63) to (68), we see that the leading-order interactions between incident fields and heterogeneities are expressed in three-dimensional volume integrals. Also the scattered and incident wavenumbers are coupled with each other. So the computation of these equations is still intensive. In this section, the parts of the integration over z in the equations are analytically estimated. Assume that the slab for each marching step is thin enough that the parameters (velocity and density) can be approximately taken as invariant along z , the integration with respect to z in Eq. (63) can be calculated as

$$\begin{aligned} & \int_{z'}^{z_1} dz e^{ik_z^\alpha(z^*-z)+i\gamma'_\alpha(z-z')} \\ &= \begin{cases} \Delta z e^{i(\gamma_\alpha+\gamma'_\alpha)\Delta z/2} \text{sinc}\left[\frac{\gamma_\alpha-\gamma'_\alpha}{2}\Delta z\right] & \text{for forescattering } (z^* = z_1), \\ \Delta z e^{i(\gamma_\alpha+\gamma'_\alpha)\Delta z/2} \text{sinc}\left[\frac{\gamma_\alpha+\gamma'_\alpha}{2}\Delta z\right] & \text{for backscattering } (z^* = z'). \end{cases} \end{aligned} \quad (69)$$

We see that the integration over z has been done analytically; however, γ_α and γ'_α are still coupled, which prevents the fast computation of the thin-slab method. To decouple γ_α and γ'_α , we neglect the angular variation of amplitude factors but keep the phase information untouched by taking the approximation $\gamma_\alpha = \gamma'_\alpha = k_\alpha$ for the amplitude factors in Eq. (69). This assumption is valid for the case where the small-angle scattering is dominant, and therefore the direction of the scattered waves are not far from the incident direction. Under this approximation, Eq. (69) becomes

$$\begin{aligned} & \int_{z'}^{z_1} dz e^{ik_z^\alpha(z^*-z)+i\gamma'_\alpha(z-z')} \\ &\approx \begin{cases} \Delta z e^{i(\gamma_\alpha+\gamma'_\alpha)\Delta z/2} & \text{for forescattering } (z^* = z_1), \\ \Delta z e^{i(\gamma_\alpha+\gamma'_\alpha)\Delta z/2} \text{sinc}(k_\alpha \Delta z) & \text{for backscattering } (z^* = z'). \end{cases} \end{aligned} \quad (70)$$

For the scattered fields P-S, S-P and S-S, similar approximations can be obtained as follows. For P-S or S-P scattering,

$$\begin{aligned} & \int_{z'}^{z_1} dz e^{ik_z^\beta(z^*-z)+i\gamma'_\alpha(z-z')} \\ &\approx \begin{cases} \Delta z e^{i(\gamma'_\alpha+\gamma_\beta)\Delta z/2} \text{sinc}[(k_\alpha - k_\beta)\Delta z/2] & \text{for forescattering } (z^* = z_1), \\ \Delta z e^{i(\gamma'_\alpha+\gamma_\beta)\Delta z/2} \text{sinc}[(k_\alpha + k_\beta)\Delta z/2] & \text{for backscattering } (z^* = z'). \end{cases} \end{aligned} \quad (71)$$

For S–S scattering,

$$\int_{z'}^{z_1} dz e^{ik_z^\beta(z^*-z)+i\gamma_\beta'(z-z')} \approx \begin{cases} \Delta z e^{i(\gamma_\beta+\gamma_\beta')\Delta z/2} & \text{for forescattering } (z^* = z_1), \\ \Delta z e^{i(\gamma_\beta+\gamma_\beta')\Delta z/2} \text{sinc}(k_\beta \Delta z) & \text{for backscattering } (z^* = z'). \end{cases} \quad (72)$$

After integration over z , the integration over transverse plane $\mathbf{x}_T$ in Eqs. (63)–(68) can be carried out by the FFT. In order to further expedite the computation, we can group the scattered field equations (63) to (68) into $\mathbf{U}^P(\mathbf{K}_T, z^*) = \mathbf{U}^{PP}(\mathbf{K}_T, z^*) + \mathbf{U}^{SP}(\mathbf{K}_T, z^*)$, and $\mathbf{U}^S(\mathbf{K}_T, z^*) = \mathbf{U}^{PS}(\mathbf{K}_T, z^*) + \mathbf{U}^{SS}(\mathbf{K}_T, z^*)$, i.e.,

$$\begin{aligned} \mathbf{U}^P(\mathbf{K}_T, z^*) &= \frac{ik_\alpha^2}{2\gamma_\alpha} e^{i\gamma_\alpha \Delta z/2} \Delta z \hat{k}_\alpha \\ &\times \left\{ \hat{k}_\alpha \cdot \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{\delta\rho(\mathbf{x}_T)}{\rho} [\eta^{PP} \mathbf{u}_\alpha^f(\mathbf{x}_T) + \eta^{SP} \mathbf{u}_\beta^f(\mathbf{x}_T)] \right. \\ &- \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{\delta\lambda(\mathbf{x}_T)}{\lambda + 2\mu} \frac{1}{ik_\alpha} \nabla \cdot [\eta^{PP} \mathbf{u}_\alpha^f(\mathbf{x}_T)] \\ &- (\hat{k}_\alpha \hat{k}_\alpha) : \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{2\delta\mu(\mathbf{x}_T)}{\lambda + 2\mu} \frac{1}{ik_\alpha} \\ &\left. \times [\eta^{PP} \varepsilon_\alpha^f(\mathbf{x}_T) + \eta^{SP} \varepsilon_\beta^f(\mathbf{x}_T)] \right\}, \end{aligned} \quad (73)$$

$$\begin{aligned} \mathbf{U}^S(\mathbf{K}_T, z^*) &= \frac{ik_\beta^2}{2\gamma_\beta} e^{i\gamma_\beta \Delta z/2} \Delta z (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \\ &\times \left\{ \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{\delta\rho(\mathbf{x}_T)}{\rho} [\eta^{PS} \mathbf{u}_\alpha^f(\mathbf{x}_T) + \eta^{SS} \mathbf{u}_\beta^f(\mathbf{x}_T)] \right. \\ &- \hat{k}_\beta \cdot \iint d^2\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \frac{2\delta\mu(\mathbf{x}_T)}{\mu} \frac{1}{ik_\beta} \\ &\left. \times [\eta^{PS} \varepsilon_\alpha^f(\mathbf{x}_T) + \eta^{SS} \varepsilon_\beta^f(\mathbf{x}_T)] \right\}, \end{aligned} \quad (74)$$

where z^* ($z^* = z'$ or $z^* = z_1$) indicates the position of the receiver plane. The modulation factors η^{PP} , $\eta^{SP} = \eta^{PS}$ and η^{SS} are

$$\eta^{PP} = \begin{cases} 1 & \text{for forescattering,} \\ \text{sinc}(k_\alpha \Delta z) & \text{for backscattering,} \end{cases} \quad (75)$$

$$\eta^{PS} = \begin{cases} \text{sinc}[(k_\alpha - k_\beta)\Delta z/2] & \text{for foreshattering,} \\ \text{sinc}[(k_\alpha + k_\beta)\Delta z/2] & \text{for backscattering,} \end{cases} \quad (76)$$

$$\eta^{SS} = \begin{cases} 1 & \text{for foreshattering,} \\ \text{sinc}(k_\beta \Delta z) & \text{for backscattering.} \end{cases} \quad (77)$$

Note that the factors $e^{i\gamma_\alpha(z-z')}$ and $e^{i\gamma_\beta(z-z')}$ have been replaced by $e^{i\gamma_\alpha\Delta z/2}$ and $e^{i\gamma_\beta\Delta z/2}$ for calculating the background fields. The phase matching (asymptotic matching) has been applied in Eqs. (73) and (74).

Under the above approximations, the thin-slab formulas may be implemented by the procedures as used in the complex screen method (Wu, 1994; Xie and Wu, 2001). First, the whole medium is sliced into appropriate thin-slabs along the overall propagation direction. Weak scattering conditions hold for each thin-slab and the parameters can be considered invariant within each thin-slab in the preferred propagation direction. Suppose that all incident fields, at the entrance of each thin-slab, are given in the wavenumber domain. The implementation procedures may be summarized as follows:

1. Freely propagate in the wavenumber domain and calculate the primary fields, the divergence of incident P wave, and strains (Eqs. (64) and (68)).
2. Inverse-FFT the primary fields, the divergence and strains into the space domain, and then calculate the distorted fields (the space domain functions before the FT in Eqs. (73)–(74)).
3. Calculate the foreshattered fields at the thin-slab exit, and add the foreshattered fields to the primary fields to form the total fields as the incident fields for the next thin-slab. In the same time backscattered fields at the thin-slab entrance are also calculated.
4. Continue the procedures 1 to 3 iteratively until the last thin-slab, and the total transmitted fields are obtained at the exit of the last thin-slab.
5. Propagate back the backscattered fields from the last to the first thin-slabs using a similar iterative procedure as step 1 to 3, and sum up all backscattered fields generated by each thin-slab to get the total reflected fields at the surface.

Let us estimate the computation speed. Most calculations involve only fast Fourier transforms. The total computation time can be estimated from the time used in calling FFTs. Taking the complex screen method as a reference, for a 2-D case, the thin-slab method needs 11 inverse FFTs and 11 forward FFTs, while the complex screen method needs 5 inverse FFTs and 7 forward FFTs. For a 3-D case, the thin-slab method needs 19 inverse FFTs and 19 forward FFTs, while the complex screen method needs 7 inverse FFTs and 10 forward FFTs. The thin-slab method takes about twice as much time as the complex screen method does, but is still much faster than the full wave methods. We will see a comparison of computation time between the thin-slab method and finite difference method in Section 4.4.

4.2. Incorporation of Boundary Transmission/Reflection into the One-Return Method

For the one-return elastic wave propagators mentioned above, the boundary transmission and reflection for a thick-layer (much thicker than the wavelength) is formed by the superposition and interference of numerous thin-slab scatterings. This gives the flexibility of the thin-slab propagator to treat arbitrarily irregular interface. However, for a homogeneous thick-layer, the calculation of boundary reflection/transmission (R/T) using the reflectivity method (see Aki and Richards, Chapter 5, 1980) is very efficient and accurate. Therefore the reflectivity method has been incorporated into the thin-slab propagator (Wu and Wu, 2003a, 2003b). In addition to the increase of efficiency, this can also reduce the accumulated errors of the thin-slab propagator when propagating in a thick layer with strong contrast of parameters from the surrounding medium. Since the thin-slab method is based on a perturbation approach, the choice of the background medium is an important issue. To make the perturbation small, the background medium parameters are changed as soon as the waves enter a laterally homogeneous region. Take the model shown on Fig. 10 as an example. On top of the irregular structure (with grey color) is a homogeneous background. When the wave enters the laterally heterogeneous part of the medium, the thin-slab propagator is used for the propagation. We can put in an artificial boundary as shown in the model with bold lines. At that artificial boundary, the reference parameters will jump from the top medium to the grey structure, a -10% jump. The reflection/transmission at that boundary can be calculated using the analytical formulation, such as the Zoeppritz equation, and then the propagation can be done, with an elastic propagator in homogeneous media, to the bottom by one big step. Note that the artificial boundary is added only to facilitate the calculation. The reflected field generated by the reflectivity calculation will be cancelled by the accumulated thin-slab scattered field. Therefore the artificial boundary will not generate spurious arrivals.

The thin-slab propagator is a dual-domain (space-wavenumber domains) approach. In a wavenumber domain, the wavefield can be expressed by a superposition of plane waves. Therefore, incorporating a reflectivity calculation into the thin-slab propagator is straightforward. After plane wave decomposition, the spectra of incident fields can be decomposed into P- and SV- and SH-components, and the R/T formulation can be applied directly to these components. In Section 4.4 numerical examples will be given to show the validity and efficiency of this approach.

4.3. Treatment of Anelasticity: The Q-Factor

Spatially varying quality factors (Q_P and Q_S) can also be incorporated into the thin-slab propagator to study the effects of intrinsic attenuation in visco-elastic media. Spatially varying intrinsic attenuation can cause not only wavefield at-

tenuation but also scattering and frequency-dependent reflections. The thin-slab propagator, with spatially varying Q -factor, is an efficient tool for such study.

For elastic, isotropic media, Lamé constants λ , μ , λ_0 , and μ_0 are related with elastic parameters by

$$\lambda = \rho\alpha^2 - 2\rho\beta^2, \quad \mu = \rho\beta^2, \quad (78)$$

$$\lambda_0 = \rho_0\alpha_0^2 - 2\rho_0\beta_0^2, \quad \mu_0 = \rho_0\beta_0^2, \quad (79)$$

where α_0 , and β_0 are compressional- and shear-wave velocities of background medium. We introduce complex velocities by performing the following transforms:

$$\alpha \rightarrow \alpha(1 - i/2Q_P), \quad \beta \rightarrow \beta(1 - i/2Q_S), \quad (80)$$

$$\alpha_0 \rightarrow \alpha_0(1 - i/2Q_P^0), \quad \beta_0 \rightarrow \beta_0(1 - i/2Q_S^0), \quad (81)$$

where Q_P^0 and Q_S^0 are compressional- and shear-wave quality factors of background medium. i is the imaginary unit. Once all parameters in Eqs. (80)–(81) are known, we can calculate the perturbations $\delta\lambda$ and $\delta\mu$ using Eqs. (78)–(79). Now λ_0 , and μ_0 become complex. As a result, the reference wavenumbers of P- and S-waves also become complex. The heterogeneities of quality factors (Q_P and Q_S) have been included in the complex $\delta\lambda$ and $\delta\mu$. With the above extension, the dual-domain thin-slab propagators can be used to model visco-elastic seismic responses. In Section 6.3 some numerical examples will be given.

4.4. Numerical Tests for the Elastic Thin-Slab Method

4.4.1. Reflection Coefficient Calculations

The following examples show the angle dependence of amplitudes (reflection coefficients) calculated by the thin-slab method. The model used is defined on a 2048×200 rectangular grid. The grid spacing in the horizontal direction is 16 m and in the vertical direction is 4 m. A horizontal interface is introduced in the middle of the model. The upper layer has $\alpha = 3.6$ km/s, $\beta = 2.08$ km/s and $\rho = 2$ g/cm³, which is taken as background medium. The lower layer has different P and S wave velocities relative to the upper layer. A 15 Hz plane P wave (or S wave) is incident on the interface at different angles. To enhance the stability in the calculation of reflection coefficients, we chose 500 samples (displacement amplitudes in the space domain) at the center of the model for both the incident and reflected fields and calculate their means, respectively. Reflection/conversion coefficients are defined by the ratio of the reflected amplitudes to the incident amplitudes at the same receiver plane. Figure 4 shows the reflection/conversion coefficients versus angle of incidence with a perturbation of 10% for both P and S wave velocities. The theoretical reflection/conversion coefficients (dashed lines)

FIG. 4. Comparisons of reflection coefficients calculated by the thin-slab method and the exact solutions. The upper panel corresponds to P wave and the lower panel to S wave incidences respectively. The bottom layer of the model has 10% velocity perturbations for both P and S waves with respect to the top layer.

are also given as references. The upper panel corresponds to P wave incidence and the lower panel, S wave incidence. The angles of incidence of S wave are limited to the critical angle of S-P converted wave. Both results agree well with the theory for a wide range of incident angles (55° for P wave incidence, and near critical angle 32° for S wave incidence).

Figures 5 to 7 show similar results as shown in Fig. 4 but with different perturbations of -10% , 20% and -20% , respectively. Figure 5 corresponds to a negative velocity perturbation. For P wave incidence, no critical angle exists. However, errors occur for large angles of incidence ($>65^\circ$ for P wave incidence). This is limited by the ability of the one-way propagator to handle wide-angle waves. For S wave incidence, both results are in good agreement up to the critical angle. Figure 6 corresponds to a perturbation of 20% for both P and S wave velocities. For a small angle of incidence (40° for P wave incidence and 20° for S wave incidence), the thin-slab results match the theoretical values. Comparing Figs. 6 and 4, we see that the wide-angle capacity of the thin-slab method decreases as perturbations increase. Figure 7 corresponds to a perturbation of -20% for both P and S wave velocities.

FIG. 5. Comparisons of reflection coefficients calculated by the thin-slab method and the exact solutions. The perturbation of the bottom layer is -10% with respect to the top layer for both P and S wave velocities.

4.4.2. 2-D Synthetic Seismograms

Synthetic seismograms are also generated to further demonstrate the capability of the thin-slab method. Figure 8 shows a 2-D model that is a vertical slice cut from the elastic French model (French, 1974). The model has a strong irregular interface that will generate large-angle reflections and scattering. The parameters of the background medium are taken as $\alpha_0 = 3.6$ km/s, $\beta_0 = 2.08$ km/s and $\rho_0 = 2.2$ g/cm³. The layer in black color has a perturbation of -20% for both P and S wave velocities. Source and receiver geometry are also shown in the figure. A Ricker wavelet with a dominant frequency of 20 Hz is used. For the thin-slab method, the spacing interval is 8 m in both horizontal and vertical directions. For the finite difference method, the spacing interval is 4 m and time interval is 0.5 ms. The stability criterion is satisfied. The direct arrivals have been properly removed from the finite difference results. Figure 9 shows a comparison of the synthetic seismograms between the thin-slab method and finite difference method. Both amplitudes and arrival times are in good agreement up to large offsets (~ 1.4 km) compared to the depth (~ 1 km). For this

FIG. 6. Comparisons of reflection coefficients calculated by the thin-slab method and the exact solutions. The perturbation of the bottom layer is 20% with respect to the top layer for both P and S wave velocities.

example, the thin-slab method is about 57 times faster than the finite difference method. The factor will increase as the size of model increases, especially for 3-D cases.

4.4.3. 3-D Synthetic Seismograms

Figure 10 shows a 3-D French model. The parameters of the background medium are taken as $\alpha_0 = 3.6$ km/s, $\beta_0 = 2.08$ km/s and $\rho_0 = 2.2$ g/cm³. The Grey structure has a perturbation of -10% for both P- and S-wave velocities. A Ricker wavelet with a dominant frequency of 10 Hz is used. For the 8th-order 3-D elastic finite-difference method (Yoon and McMechan, 1996), the spacing interval is 20 m. The actual grid size used is $250 \times 250 \times 250$ including 25 grids of absorbing boundary for each face of the model. Time interval used is 0.001 s and 2500 time steps are calculated. It took about 28 hours. For thin-slab method, the spacing interval used is 20 m in transversal plane, which is the same as used for the finite-difference method. But a fine grid size of 5 m is used in propagation direction. We did the same calculation on the same machine using the

FIG. 7. Comparisons of reflection coefficients calculated by the thin-slab method and the exact solutions. The perturbation of the bottom layer is -20% with respect to the top layer for both P and S wave velocities.

FIG. 8. A 2-D heterogeneous model (French, 1974) with irregular interface used to test the validity and accuracy of the thin-slab method. The background medium has $\alpha_0 = 3.6$ km/s, $\beta_0 = 2.08$ km/s and $\rho_0 = 2.2$ g/cm³. The layer in black color has a perturbation of -20% for both P and S wave velocities.

FIG. 9. Comparison of synthetic seismograms calculated by finite difference (solid) and by thin-slab methods (dashed). Curves in (a) and (b) are horizontal and vertical components of displacement, respectively.

thin-slab propagator. It took 2.7 hours. Thin-slab is about 10 times faster than the finite-difference method. Figure 11 gives the 3 components of the synthetic seismograms calculated using the finite-difference method (solid) and by the thin-slab method (dotted). The Y-component in Fig. 11 has been multiplied by a factor of 3. We see that the results of the two methods are in excellent agreements for small to mild angle scatterings. Especially for the reflected/converted waves generated

FIG. 10. The 3-D French model for the numerical tests. The fat solid lines delineated the horizontal surface at which the thin-slab method is connected to the reflectivity method. The background medium has the same parameters as those in Fig. 8. The structure in grey color has a perturbation of -10% for both P and S velocities.

at the lower interface of the model, the use of reflectivity method improved the accuracy of the arrival times of those events, and matched well with those by the finite-difference method.

5. THE SCREEN APPROXIMATION

For some special applications, such as slowly varying media, the synthetics only involve small-angle scattering. In this case the screen approximation can be applied to accelerate the computation.

Under small-angle scattering approximation, we can compress the thin-slab into an equivalent screen and therefore change the 3-D spectrum into a 2-D spectrum. Dual-domain implementation of the screen approximation will make the modeling of backscattering very efficient. In the following, the cases of acoustic and elastic media will be given respectively.

5.1. Screen Propagators for Acoustic Media

5.1.1. Thin-Slab Formulation in Wavenumber Domain

In order to further accelerate the computation, approximations to the interaction between the thin-slab and incident waves can be applied. First, we discuss

FIG. 11. Comparison of synthetic seismograms calculated by finite difference (solid lines) and by thin-slab methods (dashed lines). From top to the bottom are X -, Y - and Z -components of displacement, respectively.

the thin-slab formulation in wavenumber domain, and in the next section, the screen approximation will be made based on the wavenumber domain formulation.

To obtain the wavenumber domain formulation, we carry out analytically the integration along z -direction between the slab entrance z' and the exit z_1 (see Fig. 3). In the case of acoustic media, we substitute (50) and (51) into Eq. (48) and perform the moving frame coordinate transform $z \rightarrow z - z'$, resulting in

$$\begin{aligned}
P(\mathbf{K}_T, z^*) &= \frac{i}{2\gamma} \frac{k^2}{4\pi^2} e^{i(\pm\gamma)(z^*-z')} \iint d\mathbf{K}'_T \\
&\times \left\{ \int_0^{\Delta z} dz \iint d^2\mathbf{x}_T \varepsilon_\kappa(z, \mathbf{x}_T) p^0(\mathbf{K}'_T) e^{-i(\pm\gamma-\gamma')z} e^{-i(\mathbf{K}_T-\mathbf{K}'_T)\cdot\mathbf{x}_T} \right. \\
&- (\hat{k} \cdot \hat{k}') \int_0^{\Delta z} dz \iint d^2\mathbf{x}_T \varepsilon_\rho(z, \mathbf{x}_T) p^0(\mathbf{K}'_T) e^{-i(\pm\gamma-\gamma')z} \\
&\times \left. e^{-i(\mathbf{K}_T-\mathbf{K}'_T)\cdot\mathbf{x}_T} \right\}, \tag{82}
\end{aligned}$$

where

$$p^0(\mathbf{K}'_T) = p^f(z', \mathbf{K}'_T) \tag{83}$$

is the incident field at the slab entrance, $\Delta z = z_1 - z'$, and $\pm\gamma$ correspond to foreshattering and backscattering respectively. Note that

$$\int_0^{\Delta z} dz \iint d^2\mathbf{x}_T F(\mathbf{x}) e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}} = \tilde{F}(\mathbf{k} - \mathbf{k}'), \tag{84}$$

where $\tilde{F}(\mathbf{k})$ is the 3-D Fourier transform of the thin-slab, i.e., the Slab-Spectrum, $\mathbf{k}'$ is the incident wavenumber (52), and $\mathbf{k}$ is the outgoing wavenumber (scattering wavenumber) defined as

$$\mathbf{k} = \mathbf{k}^f = \mathbf{K}_T + \gamma \hat{e}_z \tag{85}$$

for foreshattered field and

$$\mathbf{k} = \mathbf{k}^b = \mathbf{K}_T - \gamma \hat{e}_z \tag{86}$$

for backscattered field. Therefore the local Born scattering in wavenumber domain can be written as

$$\begin{aligned}
P(\mathbf{K}_T, z^*) &= i \frac{k^2}{8\pi^2\gamma} e^{ik_z(z^*-z')} \\
&\times \iint d\mathbf{K}'_T [\varepsilon_\kappa(\mathbf{K}_T - \mathbf{K}'_T, k_z - \gamma') \\
&- (\hat{k} \cdot \hat{k}') \varepsilon_\rho(\mathbf{K}_T - \mathbf{K}'_T, k_z - \gamma')] p^0(\mathbf{K}'_T), \tag{87}
\end{aligned}$$

with $k_z = \gamma$ for foreshattering and $k_z = -\gamma$ for backscattering. When the receiving level is at the bottom of the thin-slab (foreshattering), $z^* = z_1$; while $z^* = z'$ is for the backscattered field at the entrance of the thin-slab. The total transmitted field at the slab exit can be calculated as the sum of the foreshattered field and the

primary field, which can be approximated as

$$p^0(\mathbf{x}'_T, z^*) = \frac{1}{4\pi^2} \iint d\mathbf{K}_T p^0(\mathbf{K}_T) e^{i\gamma(z^* - z')} e^{i\mathbf{K}_T \cdot \mathbf{x}'_T}. \quad (88)$$

We see that the scattering characteristics depend on the spectral properties of heterogeneities. In the case of large-scale heterogeneities, where lateral sizes of heterogeneities are much larger than wavelength, the major forescattered energy is concentrated in a small cone towards the forward direction. For forescattering, the outgoing wavenumbers $\mathbf{k}$ are nearly in the same direction as the incoming wavenumbers $\mathbf{k}'$. Within the small cone, $\mathbf{k} - \mathbf{k}'$ stays small. Therefore, the scattered waves are controlled by the low lateral spatial-frequency components of heterogeneities. In the meanwhile, $\mathbf{k}$ and $\mathbf{k}'$ have opposite directions for backscattered waves and the backscattering is most sensitive to those vertical spectral components which are comparable to the half-wavelength (see Wu and Aki, 1985). Also, we know that forescattering is controlled by velocity heterogeneities, while backscattering responses mostly to impedance heterogeneities (ibid). This point will be seen much more clearly in the next section.

Note that the wave-slab interaction in wavenumber domain [Eq. (87)] is not a convolution and therefore the operation in space domain is not local. Therefore, the wave-slab interaction in wavenumber domain involves matrix multiplication and is computationally intensive.

5.1.2. Small-Angle Approximation and the Screen Propagators

Under this approximation, both incoming and outgoing wavenumbers have small transversal components K_T compared to the longitudinal component γ and therefore

$$\gamma = \sqrt{k^2 - K_T^2} \approx k \left(1 - \frac{K_T^2}{2k} \right). \quad (89)$$

Then

$$\begin{aligned} \mathbf{k}^f - \mathbf{k}' &= (\mathbf{K}_T - \mathbf{K}'_T) + (\gamma - \gamma') \hat{e}_z \\ &\approx (\mathbf{K}_T - \mathbf{K}'_T) + \left(\frac{K_T'^2}{2k} - \frac{K_T^2}{2k} \right) \hat{e}_z \\ &\approx (\mathbf{K}_T - \mathbf{K}'_T) + 0 \hat{e}_z \end{aligned} \quad (90)$$

and

$$\mathbf{k}^b - \mathbf{k}' = (\mathbf{K}_T - \mathbf{K}'_T) - (\gamma + \gamma') \hat{e}_z \approx (\mathbf{K}_T - \mathbf{K}'_T) - 2k \hat{e}_z. \quad (91)$$

5.1.3. Screen Approximation in Acoustic Media

With the small-angle approximation, the 3-D thin-slab spectrum (84) can be approximated by

$$\begin{aligned}
 \tilde{F}(\mathbf{k}^f - \mathbf{k}) &\approx \tilde{F}(\mathbf{K}_T - \mathbf{K}'_T, K_z = 0) \\
 &= \iint d^2\mathbf{x}_T e^{-i(\mathbf{K}_T - \mathbf{K}'_T) \cdot \mathbf{x}_T} \int_0^{\Delta z} dz [\varepsilon_\kappa(\mathbf{x}_T, z) - \varepsilon_\rho(\mathbf{x}_T, z)] \\
 &= 2 \iint d^2\mathbf{x}_T e^{-i(\mathbf{K}_T - \mathbf{K}'_T) \cdot \mathbf{x}_T} S_V(\mathbf{x}_T),
 \end{aligned} \tag{92}$$

where S_V is a 2-D screen of velocity perturbation, and

$$\begin{aligned}
 \tilde{F}(\mathbf{k}^b - \mathbf{k}) &\approx \tilde{F}(\mathbf{K}_T - \mathbf{K}'_T, K_z = -2k) \\
 &= \iint d^2\mathbf{x}_T e^{-i(\mathbf{K}_T - \mathbf{K}'_T) \cdot \mathbf{x}_T} \int_0^{\Delta z} dz e^{i2kz} [\varepsilon_\kappa(\mathbf{x}_T, z) + \varepsilon_\rho(\mathbf{x}_T, z)] \\
 &= 2 \iint d^2\mathbf{x}_T e^{-i(\mathbf{K}_T - \mathbf{K}'_T) \cdot \mathbf{x}_T} S_I(\mathbf{x}_T),
 \end{aligned} \tag{93}$$

where S_I is a 2-D screen of impedance perturbation. We see that with the above approximation, 3-D thin-slab spectra have been replaced by 2-D screen spectra that are slices of the 3-D spectra. In the case of forescattering, the slice is from a velocity spectrum at $K_z = 0$, where K_z is the spatial frequency in the z -direction; while for backscattering, from an impedance spectrum, a slice at $K_z = -2k$. In the special case when $F(\mathbf{x}_T, z)$ varies very little along z within the thin-slab, the screen spectra can be further approximated as

$$\begin{aligned}
 \tilde{F}(\mathbf{k}^f - \mathbf{k}) &\approx 2\tilde{S}_V(\mathbf{K}_T - \mathbf{K}'_T) = [\delta_\kappa(\mathbf{K}_T - \mathbf{K}'_T) - \delta_\rho(\mathbf{K}_T - \mathbf{K}'_T)]\Delta z, \\
 \tilde{F}(\mathbf{k}^b - \mathbf{k}) &\approx 2\tilde{S}_I(\mathbf{K}_T - \mathbf{K}'_T) \\
 &= [\delta_\kappa(\mathbf{K}_T - \mathbf{K}'_T) + \delta_\rho(\mathbf{K}_T - \mathbf{K}'_T)]\Delta z \operatorname{sinc}(k\Delta z) e^{ik\Delta z}.
 \end{aligned} \tag{94}$$

The scattered fields (74) under the screen approximation become

$$P(\mathbf{K}_T, z^*) \approx i \frac{k^2}{4\pi^2\gamma} e^{ik_z(z^* - z')} \iint d\mathbf{K}'_T \tilde{S}(\mathbf{K}_T - \mathbf{K}'_T) p^0(\mathbf{K}'_T). \tag{95}$$

The above equation is a convolution integral in wavenumber domain and the corresponding operation in space domain is a local one. The dual-domain technique

can be used to speed up the computation:

$$P(\mathbf{K}_T, z^*) \approx \frac{ik^2}{\gamma} e^{ik_z(z^*-z')} \iint d\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} S(\mathbf{x}_T) p^0(\mathbf{x}_T), \quad (96)$$

where

$$\begin{aligned} S(\mathbf{x}_T) &= S_V(\mathbf{x}_T) \\ &= \frac{1}{4} \int_0^{\Delta z} dz [\delta_\kappa(\mathbf{x}_T, z) - \delta_\rho(\mathbf{x}_T, z)] \quad \text{for foreshattering,} \end{aligned} \quad (97)$$

$$\begin{aligned} S(\mathbf{x}_T) &= S_I(\mathbf{x}_T) \\ &= \frac{1}{4} \int_0^{\Delta z} dz e^{i2kz} [\delta_\kappa(\mathbf{x}_T, z) + \delta_\rho(\mathbf{x}_T, z)] \quad \text{for backscattering.} \end{aligned} \quad (98)$$

The total transmitted field at z_1 is

$$\begin{aligned} P^f(\mathbf{K}_T, z_1) &= p^0(\mathbf{K}_T, z_1) + P^f(\mathbf{K}_T, z_1) \\ &= e^{ik_z(z_1-z')} \iint d\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} p^0(\mathbf{x}_T) [1 + ikS_V(\mathbf{x}_T)] \\ &= e^{ik_z(z_1-z')} \iint d\mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} p^0(\mathbf{x}_T) \exp[ikS_V(\mathbf{x}_T)], \end{aligned} \quad (99)$$

where $k/\gamma_\alpha \approx 1$ has been used for the scattered field based on the small-angle scattering approximation. The above equation is the dual-domain implementation of phase-screen propagation.

5.1.4. Procedure of MFSB Using the Screen Approximation

1. Fourier transform the incident field at the starting plane into wavenumber domain and propagate to the screen using a constant velocity propagator.
2. Inverse Fourier transform the incident field into space domain. Interact with the impedance screen (complex-screen) to get the backscattered field and interact with the velocity screen (phase-screen) to get the transmitted field.
3. Fourier transform the transmitted field into wavenumber domain and propagate to the next screen using a constant velocity propagator.
4. Repeat the propagation and interaction screen-by-screen to the bottom of the model space.
5. Backpropagate and stack the stored backscattered field screen by screen from the bottom to the top to get the total backscattered field on the surface.

5.2. Screen Approximation for Elastic Media: Elastic complex Screen Method

Similar to the derivation for acoustic media, the wavenumber domain formulation can be obtained by substituting Eqs. (64), (68) into Eqs. (63) and (65)–(67).

$$\begin{aligned}
 \mathbf{U}^{PP}(\mathbf{K}_T, \mathbf{K}'_T) &= \frac{i}{2\gamma_\alpha} k_\alpha^2 u_0^P \hat{k}_\alpha \\
 &\quad \times \left[(\hat{k}_\alpha \cdot \hat{k}'_\alpha) \frac{\delta\rho(\tilde{\mathbf{k}})}{\rho} - \frac{\delta\lambda(\tilde{\mathbf{k}})}{\lambda + 2\mu} - (\hat{k}_\alpha \cdot \hat{k}'_\alpha)^2 \frac{2\delta\mu(\tilde{\mathbf{k}})}{\lambda + 2\mu} \right], \\
 \mathbf{U}^{PS}(\mathbf{K}_T, \mathbf{K}'_T) &= \frac{i}{2\gamma_\beta} k_\beta^2 (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \cdot \mathbf{u}_0^P \left[\frac{\delta\rho(\tilde{\mathbf{k}})}{\rho} - 2 \frac{\beta_0}{\alpha_0} (\hat{k}_\alpha \cdot \hat{k}'_\beta) \frac{\delta\mu(\tilde{\mathbf{k}})}{\mu} \right], \\
 \mathbf{U}^{SP}(\mathbf{K}_T, \mathbf{K}'_T) &= \frac{i}{2\gamma_\alpha} k_\alpha^2 (\mathbf{u}_0^S \cdot \hat{k}_\alpha) \hat{k}_\alpha \left[\frac{\delta\rho(\tilde{\mathbf{k}})}{\rho} - 2 \frac{\beta_0}{\alpha_0} (\hat{k}_\beta \cdot \hat{k}'_\alpha) \frac{\delta\mu(\tilde{\mathbf{k}})}{\mu} \right], \\
 \mathbf{U}^{SS}(\mathbf{K}_T, \mathbf{K}'_T) &= \frac{i}{2\gamma_\beta} k_\beta^2 (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \cdot \left\{ \mathbf{u}_0^S \left[\frac{\delta\rho(\tilde{\mathbf{k}})}{\rho} - (\hat{k}_\beta \cdot \hat{k}'_\beta) \frac{\delta\mu(\tilde{\mathbf{k}})}{\mu} \right] \right. \\
 &\quad \left. - \hat{k}'_\beta (\mathbf{u}_0^S \cdot \hat{k}_\beta) \frac{\delta\mu(\tilde{\mathbf{k}})}{\mu} \right\}, \tag{100}
 \end{aligned}$$

where $\mathbf{u}_0^P$ is the spectral field of the incident P wave, and $\delta\rho(\tilde{\mathbf{k}})$, $\delta\lambda(\tilde{\mathbf{k}})$ and $\delta\mu(\tilde{\mathbf{k}})$ are the three-dimensional Fourier transforms of medium perturbations, and $\tilde{\mathbf{k}} = \hat{k} - \hat{k}'$ is the exchange wavenumber with $\hat{k}'$ and $\hat{k}$ as incident and scattering wavenumber vectors, respectively.

From the thin-slab formulation, under the small-angle approximation, both incident and scattered wavenumbers have small lateral components K_T and K'_T compared to vertical components and therefore

$$\begin{aligned}
 \gamma_\alpha &\approx k_\alpha (1 - K_T^2/2k_\alpha^2), & \gamma_\beta &\approx k_\beta (1 - K_T^2/2k_\beta^2), \\
 \gamma'_\alpha &\approx k_\alpha (1 - K_T'^2/2k_\alpha^2), & \gamma'_\beta &\approx k_\beta (1 - K_T'^2/2k_\beta^2). \tag{101}
 \end{aligned}$$

Using these approximations and neglecting higher-order (greater than second order) small quantities, the scattering patterns can be obtained as

$$\begin{aligned}
 (\hat{k}_\alpha \cdot \hat{k}'_\alpha) &\approx \begin{cases} +1 + (|\mathbf{K}_T - \mathbf{K}'_T|)^2/2k_\alpha^2 & \text{for forescattering,} \\ -1 + (|\mathbf{K}_T + \mathbf{K}'_T|)^2/2k_\alpha^2 & \text{for backscattering,} \end{cases} \\
 (\hat{k}_\alpha \cdot \hat{k}'_\beta) &\approx \begin{cases} +1 - (|k_\beta \mathbf{K}_T - k_\alpha \mathbf{K}'_T|)^2/(2k_\alpha^2 k_\beta^2) & \text{for forescattering,} \\ -1 + (|k_\beta \mathbf{K}_T + k_\alpha \mathbf{K}'_T|)^2/(2k_\alpha^2 k_\beta^2) & \text{for backscattering,} \end{cases} \\
 (\hat{k}_\beta \cdot \hat{k}'_\alpha) &\approx \begin{cases} +1 - (|k_\alpha \mathbf{K}_T - k_\beta \mathbf{K}'_T|)^2/(2k_\alpha^2 k_\beta^2) & \text{for forescattering,} \\ -1 + (|k_\alpha \mathbf{K}_T + k_\beta \mathbf{K}'_T|)^2/(2k_\alpha^2 k_\beta^2) & \text{for backscattering,} \end{cases} \\
 (\hat{k}_\beta \cdot \hat{k}'_\beta) &\approx \begin{cases} +1 + (|\mathbf{K}_T - \mathbf{K}'_T|)^2/2k_\beta^2 & \text{for forescattering,} \\ -1 + (|\mathbf{K}_T + \mathbf{K}'_T|)^2/2k_\beta^2 & \text{for backscattering.} \end{cases} \tag{102}
 \end{aligned}$$

To decouple the incident wavenumber $\mathbf{K}_T$ and the scattered wavenumber $\mathbf{K}'_T$ in Eqs. (102), suppose that the medium heterogeneities are smooth enough that the scattering wavenumbers $\hat{k}_\alpha$ or $\hat{k}_\beta$ deviates not too far from the incident wavenumbers $\hat{k}'_\alpha$ or $\hat{k}'_\beta$. We can take $\mathbf{K}'_T$ to be approximately equal to $\mathbf{K}_T$, the wavenumber couplings may be simplified further by

$$\begin{aligned} (\hat{k}_\alpha \cdot \hat{k}'_\alpha) &\approx \begin{cases} +1 & \text{for forescattering,} \\ -1 + 2K_T^2/k_\alpha^2 & \text{for backscattering,} \end{cases} \\ (\hat{k}_\alpha \cdot \hat{k}'_\beta) &\approx \begin{cases} +1 - (K_T/k_\beta - K_T/k_\alpha)^2/2 & \text{for forescattering,} \\ -1 + (K_T/k_\beta + K_T/k_\alpha)^2/2 & \text{for backscattering,} \end{cases} \\ (\hat{k}_\beta \cdot \hat{k}'_\alpha) &\approx \begin{cases} +1 - (K_T/k_\alpha - K_T/k_\beta)^2/2 & \text{for forescattering,} \\ -1 + (K_T/k_\alpha + K_T/k_\beta)^2/2 & \text{for backscattering,} \end{cases} \\ (\hat{k}_\beta \cdot \hat{k}'_\beta) &\approx \begin{cases} +1 & \text{for forescattering,} \\ -1 + 2K_T^2/k_\beta^2 & \text{for backscattering.} \end{cases} \end{aligned} \quad (103)$$

It can be seen from Eqs. (102)–(103) that the first-order corrections have relatively stronger effects on the backscattering than on the forescattering. The scattering pattern $(\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \cdot \hat{k}'_\beta$ appearing in Eqs. (103) is zero for forward scattering, while for backward scattering it is

$$\begin{aligned} [(\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \cdot \hat{k}'_\beta]_{\text{backward}} &= [\hat{k}'_\beta - (\hat{k}_\beta \cdot \hat{k}'_\beta) \hat{k}_\beta]_{\text{backward}} \\ &\approx [\hat{k}'_\beta - (-1 + 2K_T^2/k_\beta^2) \hat{k}_\beta]_{\text{backward}} \\ &\approx (2K_T/k_\beta, 2K_T^2/k_\beta^2), \end{aligned} \quad (104)$$

up to the first-order correction.

5.3. Complex Screen Method with First-Order Corrections

Numerical tests show that the effect of the first-order corrections of wave couplings for P–S and S–P forescattering can be neglected. So the first-order corrections are introduced only for backward propagators. Substituting Eqs. (103)–(104) into Eqs. (100) and integrating over incident wavenumber $\hat{k}'$, we can obtain the following formulas

$$\mathbf{U}_f^{PP}(\mathbf{K}'_T, \mathbf{K}_T) = -ik_\alpha \Delta z \hat{k}'_\alpha u_0^P(\mathbf{K}_T) \frac{\delta \alpha(\tilde{\mathbf{K}}_T)}{\alpha_0} \eta_f^{PP}, \quad (105)$$

$$\begin{aligned} \mathbf{U}_f^{PS}(\mathbf{K}'_T, \mathbf{K}_T) &= -ik_\beta \Delta z u_0^P(\mathbf{K}_T) [\hat{k}_\alpha - \hat{k}'_\beta (\hat{k}_\alpha \cdot \hat{k}'_\beta)] \\ &\quad \times \left[\frac{\delta \beta(\tilde{\mathbf{K}}_T)}{\beta_0} + \left(\frac{\beta_0}{\alpha_0} - \frac{1}{2} \right) \frac{\delta \mu(\tilde{\mathbf{K}}_T)}{\mu_0} \right] \eta_f^{PS}, \end{aligned} \quad (106)$$

$$\begin{aligned} \mathbf{U}_f^{SP}(\mathbf{K}'_T, \mathbf{K}_T) &= -ik_\alpha \Delta z (\mathbf{u}_0^S(\mathbf{K}_T) \cdot \hat{k}_\alpha) \hat{k}_\alpha \\ &\quad \times \left[\frac{\delta \beta(\tilde{\mathbf{K}}_T)}{\beta_0} + \left(\frac{\beta_0}{\alpha_0} - \frac{1}{2} \right) \frac{\delta \mu(\tilde{\mathbf{K}}_T)}{\mu_0} \right] \eta_f^{SP}, \end{aligned} \quad (107)$$

$$\mathbf{U}_f^{SS}(\mathbf{K}'_T, \mathbf{K}_T) = -ik_\beta \Delta z [\mathbf{u}_0^S(\mathbf{K}_T) - \hat{k}'_\beta (\mathbf{u}_0^S(\mathbf{K}_T) \cdot \hat{k}'_\beta)] \frac{\delta\beta(\tilde{\mathbf{K}}_T)}{\beta_0} \eta_f^{SS}, \quad (108)$$

$$\begin{aligned} \mathbf{U}_b^{PP}(\mathbf{K}_T, z_1) &= \frac{i}{2} k_\alpha \hat{k}_\alpha \Delta z e^{ik_\alpha \Delta z} \eta_b^{PP} \\ &\times \left[\left(-1 + \frac{2K_T^2}{k_\alpha^2} \right) V_\rho - V_\lambda - \left(1 - \frac{2K_T^2}{k_\alpha^2} \right)^2 V_\mu \right], \end{aligned} \quad (109)$$

$$\begin{aligned} \mathbf{U}_b^{PS}(\mathbf{K}_T, z_1) &= \frac{i}{2} k_\beta \Delta z e^{i(k_\alpha + k_\beta) \Delta z/2} \eta_b^{PS} (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \\ &\times \left\{ \mathbf{V}_\rho^P - 2 \frac{\beta_0}{\alpha_0} \left[-1 + \frac{1}{2} \left(\frac{K_T}{k_\alpha} + \frac{K_T}{k_\beta} \right)^2 \right] \mathbf{V}_\mu^P \right\}, \end{aligned} \quad (110)$$

$$\begin{aligned} \mathbf{U}_b^{SP}(\mathbf{K}_T, z_1) &= \frac{i}{2} k_\alpha \Delta z e^{i(k_\alpha + k_\beta) \Delta z/2} \eta_b^{SP} \hat{k}_\alpha \hat{k}_\alpha \\ &\times \left\{ \mathbf{V}_\rho^S - 2 \frac{\beta_0}{\alpha_0} \left[-1 + \frac{1}{2} \left(\frac{K_T}{k_\alpha} + \frac{K_T}{k_\beta} \right)^2 \right] \mathbf{V}_\mu^S \right\}, \end{aligned} \quad (111)$$

$$\begin{aligned} \mathbf{U}_b^{SS}(\mathbf{K}_T, z_1) &= \frac{i}{2} k_\beta \Delta z e^{ik_\beta \Delta z} \eta_b^{SS} \\ &\times \left\{ (\mathbf{I} - \hat{k}_\beta \hat{k}_\beta) \cdot \left[\mathbf{V}_\rho^S + \left(1 - \frac{2K_T^2}{k_\beta^2} \right) \mathbf{V}_\mu^S \right] - \tilde{K}_\beta (\hat{k}_\beta \cdot \mathbf{V}_\mu^S) \right\}, \end{aligned} \quad (112)$$

where V_ρ is the distorted P wave amplitude by density perturbation, $\mathbf{V}_\rho^P$ is the distorted P wave vector field (displacement) by density perturbation, etc., defined as

$$\begin{aligned} V_\rho(\mathbf{K}_T) &= \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} u_0^P(\mathbf{x}_T, z') \frac{\delta\rho(\mathbf{x}_T)}{\rho}, \\ V_\lambda(\mathbf{K}_T) &= \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} u_0^P(\mathbf{x}_T, z') \frac{\delta\lambda(\mathbf{x}_T)}{\lambda + 2\mu}, \\ V_\mu(\mathbf{K}_T) &= \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} u_0^P(\mathbf{x}_T, z') \frac{2\delta\mu(\mathbf{x}_T)}{\lambda + 2\mu}, \\ \mathbf{V}_\rho^P(\mathbf{K}_T) &= \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \mathbf{u}_0^P(\mathbf{x}_T, z') \frac{\delta\rho(\mathbf{x}_T)}{\rho}, \\ \mathbf{V}_\mu^P(\mathbf{K}_T) &= \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \mathbf{u}_0^P(\mathbf{x}_T, z') \frac{\delta\mu(\mathbf{x}_T)}{\mu}, \\ \mathbf{V}_\rho^S(\mathbf{K}_T) &= \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \mathbf{u}_0^S(\mathbf{x}_T, z') \frac{\delta\rho(\mathbf{x}_T)}{\rho}, \\ \mathbf{V}_\mu^S(\mathbf{K}_T) &= \iint d^2 \mathbf{x}_T e^{-i\mathbf{K}_T \cdot \mathbf{x}_T} \mathbf{u}_0^S(\mathbf{x}_T, z') \frac{\delta\mu(\mathbf{x}_T)}{\mu}, \\ \tilde{K}_\beta(K_T) &= (2\mathbf{K}_T/k_\beta, 2K_T^2/k_\beta^2), \end{aligned}$$

and the factors η_f^{PP} , η_b^{PP} , η_f^{PS} , η_b^{PS} , η_f^{SP} , η_b^{SP} and η_f^{SS} , η_b^{SS} are given by Eqs. (75)–(77). In the above derivation, we assume that the thin-slab is thin enough so that the parameters can be approximated as unchanging along the z direction, and we also approximate γ_α and γ_β in the denominators of Eqs. (100) by k_α and k_β , respectively. Although Eqs. (109)–(112) look a little more complicated in form than those in zero-order method, they can all be implemented using the FFT and the efficiency is not compromised too much. It has been shown that the zero-order complex screen results agree with the thin-slab results only for small incidence angles. As incidence angle increases, the amplitude of the reflected waves deviates gradually from that by the thin-slab method. However, the first order complex screen results are in good agreement with the thin-slab results for fairly wide angles (40° for P wave incidence and 20° for S wave incidence for the French model).

6. REFLECTED WAVE FIELD MODELING USING THIN-SLAB METHOD

AVO analysis plays an important role in modern seismic data interpretation in exploration seismology. AVO measures the angle-dependent reflection response of an interface and relates the response to in-situ elastic parameters and fluid contents of the target intervals. For flat interfaces in homogeneous elastic media, AVO curves can be easily calculated theoretically. However, the observed reflection responses of a seismic target are significantly affected by many other factors, such as data collection, data processing and wave propagation effects in heterogeneous media. Before analyzing the AVO responses, these effects should be studied and compensated.

Forward modeling can be useful in understanding wave propagation effects on AVO. In addition, forward modeling can also be useful in interpreting complicated AVO measurements, providing appropriate model parameters for data processing, and developing algorithms of frequency-dependent AVO inversion (Dey-Sarkar and Svatek, 1993). There are several modeling algorithms available for AVO calculation. The reflectivity method is one of the most common methods used for modeling AVO responses in layered media (Fuchs and Müller, 1971; Simmons and Backus, 1994; Wapenaar *et al.*, 1999). It can generate an exact AVO response in arbitrarily layered medium, but cannot be used in structures with lateral velocity variations. Ray methods based on various approximations of the Zoeppritz equations are also very common for AVO analysis (Widess, 1973; Simmons and Backus, 1994; Bakke and Ursin, 1998). However, in the presence of thin layers, primaries-only Zoeppritz modeling may produce incorrect results. Another intrinsic limit of the ray methods is its inability in dealing with frequency-dependent scattering associated with heterogeneities. A number of authors applied pseudospectral and finite-difference methods to more complicated geologic

models including anelasticity, overburden structure, scattering attenuation, and anisotropy (Chang and McMechan, 1996; Adriansyah and McMechan, 1998; Yoon and McMechan, 1996). In principle, these methods can deal with arbitrarily complicated geologic models. However, they are very time-consuming and memory demanding especially for 3-D models. This is also true for handling structures with thin layers where fine grids must be used.

In this section we apply the elastic thin-slab method to AVO modeling in sedimentary rocks. Several numerical experiments, including reflection coefficient calculations, reflection synthetics with lateral parameter variations, and thin-bed AVO, have been conducted and compared with reflectivity and finite-difference methods. The accuracy and wide-angle capacity of the thin-slab method are demonstrated. Some examples showing the effects of lateral structure variations and random heterogeneities on AVO in sedimentary rocks are presented and analyzed.

6.1. Reflection Coefficients of Sedimentary Interfaces

Reflection coefficients vary as a function of offset from the source (or reflection angle) across an interface. This information is the core of AVO analysis. For either forward modeling or inversion, accurate calculation of reflection coefficients is crucial. Here, we use the thin-slab method to calculate the reflection coefficients at the shale/gas, shale/oil and shale/brine interfaces. A rectangular grid of 1024×200 is used in the calculation with its parameters given in Table 1. The grid spacings used are 16 m in horizontal direction and 4 m in vertical direction. The interface is located at the depth of 200 m. A taper function is applied to the bottom of the model for eliminating the reflection from the artificial truncation at bottom. Only P–P reflection coefficients are displayed. For reflection coefficient calculation, we take 200 samples (displacement amplitudes in space domain) in the middle of the model for both incident and reflected waves, and define the ratio of their average displacement amplitudes as the reflection coefficient. Figure 12 shows the P–P reflection coefficients for all sets of parameters given in Table 1. The dotted curves represent the corresponding theoretical results. ϕ is the percentage porosity, and σ_1 and σ_2 are the Poisson’s ratios for the shale and sand,

TABLE 1. Reservoir model

	$\phi = 20\%$				$\phi = 23\%$				$\phi = 25\%$			
	α	β	ρ	σ	α	β	ρ	σ	α	β	ρ	σ
Shale	3170	1608	2.36	0.31	3170	1668	2.36	0.31	3170	1668	2.36	0.31
Gas	3500	2374	2.10	0.10	3350	2231	2.12	0.10	3188	2124	1.96	0.10
Oil	3734	2280	2.27	0.20	3527	2131	2.22	0.21	3362	2015	2.18	0.22
Brine	3749	2262	2.31	0.21	3551	2109	2.27	0.23	3399	1993	2.23	0.24

FIG. 12. P-P reflection coefficients at different type of interfaces: shale/gas (top), shale/oil (middle), and shale/brine (bottom). All formation parameters are listed in Table 1. The solid curves are calculated by the thin-slab method and the dotted lines, by the Zoeppritz equations.

respectively. The top panel corresponds to shale/gas interface, the middle panel, shale/oil interface, and the bottom panel, shale/brine interface. We see that the reflection coefficients calculated by the thin-slab method are in excellent agreement with theoretical values for small and medium incident angles ($<30^\circ$) for all cases, and up to a wide angle of incidence ($<40^\circ$) for the 20-percent-porosity gas, oil, and brine sands.

Although the 20-percent-porosity sand has relatively large velocity perturbations, the thin-slab results show better matches to theoretical reflection coefficients for wide angles of incidence than those for 23-percent-porosity and 25-percent-porosity sands at wide angles. It implies that the accuracy and wide-angle

TABLE 2. Perturbations of reservoir parameter

	$\phi = 20\%$				$\phi = 23\%$				$\phi = 25\%$			
	$\frac{\delta\alpha}{\alpha} \%$	$\frac{\delta\beta}{\beta} \%$	$\frac{\delta\rho}{\rho} \%$	σ	$\frac{\delta\alpha}{\alpha} \%$	$\frac{\delta\beta}{\beta} \%$	$\frac{\delta\rho}{\rho} \%$	σ	$\frac{\delta\alpha}{\alpha} \%$	$\frac{\delta\beta}{\beta} \%$	$\frac{\delta\rho}{\rho} \%$	σ
Gas	12.3	42.3	-11.0	0.10	5.7	33.7	-14.4	0.10	0.6	27.0	-17.0	0.10
Oil	17.8	36.7	-3.80	0.20	11.3	27.8	-5.9	0.21	6.1	20.8	-7.6	0.22
Brine	18.3	35.6	-2.10	0.21	12.0	26.4	-3.8	0.23	7.2	19.5	-5.5	0.24

capacity of the thin-slab method are related not only to velocity and density perturbations but also to their combinations (the impedance).

The computational efficiency, accuracy, and wide-angle capacity of the thin-slab method are closely related to the amount of perturbations. Generally, the accuracy and wide-angle capacity decrease as perturbations (velocity and/or density) increase. Its computational efficiency also decreases because finer forward steps must be utilized to guarantee the convergence. To see the perturbations of reservoir parameters shown in Table 1, we calculate velocity and density fluctuations relative to the shale (shown in Table 2). The perturbations of the physical parameters used in the models are typical of most sedimentary rocks.

6.2. Reflections from a dipping sandstone reservoir

Figure 13 shows a model of a dipping sandstone reservoir bearing gas, oil, and brine. The dipping angle of the reservoir is 10° to horizontal plane. The reservoir is thick enough so that the reflections from the top and base are separated in seismograms. Simmons and Backus (1994) used these models to investigate AVO responses associated with angles of incidence, different type interfaces, and porosities of interest. In this section we will use these models to show the accuracy of the thin-slab method for modeling reflections including primary reflections, conversions and diffractions.

Figure 14 displays the synthetic seismograms with incident plane waves for three different porosities (labeled in each panel). The plane P-wave is vertically downward incident to the model. The left column corresponds to the finite-difference results and the right panel, to the thin-slab results. The last two arrivals are the converted shear waves produced at the top and base of the reservoir and propagated in the shale. Single-leg (one converted wave path in the layer) and double-leg (two converted wave paths in the layer) converted shear waves are weak at a small angle of incidence and overlap with the primary reflections from the base. Note that in the thin-slab results (the right column in Fig. 14) the multiples within the sandstone are neglected. Comparing the two columns, we see that the thin-slab results are in good agreement with the finite-difference results, implying that the multiples are not significant in sedimentary rocks.

FIG. 13. Model of a dipping sandstone reservoir filled with gas, oil, and brine. The reservoir is thick enough so that the reflections from the top and base can be separated.

FIG. 14. Comparisons of plane wave seismograms calculated by finite-difference (left column) and thin-slab (right column) methods for a dipping reservoir model shown in Fig. 13.

6.3. Reservoir Reflections with Scattering and Attenuation from a Heterogeneous Overburden

We examine the effect of scattering and attenuation associated with heterogeneities in sedimentary rocks on AVO using the thin-slab modeling method. First we study the effects of random scattering. The reservoir is a flat model generated by rotating the model in Fig. 13. A 2-D random field with exponential correlation function is used to perturb the velocity and density parameters of the sedimentary rocks (Fig. 15). The correlation lengths of the random field are 100 m in horizontal direction and 40 m in depth. The rms values used are 1%, 2% and 3%. Note that both P- and S-wave velocities have the same distributions and rms perturbations; while the density has the same distribution but only one half of the

FIG. 15. (a) Sandstone model with heterogeneities, (b) to (d) are Snapshots at $t = 0.2$ s, 0.4 s, and 0.6 s, respectively. A 30 Hz plane P-wave source is vertically incident from the top of the model.

rms value of velocity perturbation. Only velocity rms values are indicated in the text and figures. For simplicity, we consider a plane P-wave vertically incident on the reservoir. Figure 15 shows the model and the snapshots at $t = 0.2$ s, 0.4 s and 0.6 s, respectively. The porosity of the sand is 25% and rms velocity perturbation is 2%. In Fig. 15 we see that abundant coda waves are produced. The wavefronts of both forward and reflected waves are distorted. For the reflected waves from shale/oil and shale/brine interfaces, serious distortions can be seen. Figure 16 shows the maximum magnitudes of the responses from the top interface of the sand. The vertical axis is the logarithmic amplitude, and (a), (b) and (c) correspond to porosities of 20%, 23%, and 25%, respectively. Solid lines correspond to calculations in models without overburden heterogeneities, in which the fluctuations in amplitudes are caused by the interference of boundary diffractions. The dotted, dashed and dotted-dashed lines correspond to overburden models having rms random velocity fluctuations of 1%, 2% and 3%, respectively. The vertical interfaces separating gas, oil and brine are indicated by arrows. Note that for an overlying shale with heterogeneities as small as rms = 1%, the piece-wise uniform reflection amplitudes become fluctuating due to the focusing and defocusing of waves. As the velocity fluctuation increases, stronger amplitude fluctuations are generated. The amplitude fluctuations are closely related to the statistical properties of heterogeneities. The existence of a laterally varying overburden layer has serious effects on reflecting waves from reservoirs. It generates scattered waves and affects the reflection characteristics of local interfaces. For weak reflection sands, the scattering effect from heterogeneous overburden could be important and must be taken into account for AVO analysis. The top panel shows reservoir model bearing gas, oil and brine, respectively. The formation is anelastic and heterogeneous. The lower three panels show AVOs for various kinds of interfaces: (a) shale/gas, (b) shale/oil, and (c) shale/brine. For each kind of interface, three different constant Q 's ($Q = \text{infinity}$, 150, 50) are given to shale. The sand has constant $Q_P = Q_S = 10$. The correlation lengths of the random field for perturbing Q and elastic parameters are the same. The rms values are 4% for elastic parameters and 25% for Q .

Next, we show combined effects of random scattering and intrinsic attenuation. In practice, the geologic models may contain arbitrary spatial variations in compressional- and shear-wave quality factors, as well as density and velocities (see the top panel in Fig. 17). For each kind of interface, three different averaged Q 's (i.e., $Q = 50$, 150, infinity) are given to shale, and the sands have quality factors of $Q_P = Q_S = 10$. The correlation lengths of the random field for perturbing Q and elastic parameters are the same. The rms values are 4% for elastic parameters and 25% for Q the source and receiver array are located in shale and 1200 m away from the interface. The dotted lines in Fig. 17 correspond to the homogeneous cases with constant Q 's. We see that intrinsic attenuation mainly affect the absolute reflected amplitudes and heterogeneities in both Q and elastic parameters affect local amplitude fluctuation with offset. In summary, the AVO

FIG. 16. Effect of scattering by heterogeneities on reflected amplitudes of reservoirs shown in Fig. 15.

responses of the target subsurface have been significantly deformed due to both the heterogeneities and intrinsic attenuation.

6.4. Thin Layer AVO Response

The amplitude response of a thin-bed has drawn increasing interest in hydrocarbon interpretation because large quantities of gas reserves are found to be trapped with thin sands. The AVO response of a thin-bed is different from that of a thick bed because of the effects of wave interference, conversion and tunneling. For a thin-bed with thickness much less than the predominant wavelength, the grid methods such as finite-difference and finite-element methods are too costly for

FIG. 17. The top panel shows reservoir model bearing gas, oil and brine, respectively. The formation is anelastic and heterogeneous. The lower three panels show AVOs for various kinds of interfaces: (a) shale/gas, (b) shale/oil, and (c) shale/brine. For each kind of interface, three different constant Q 's ($Q = \infty, 150, 50$) are given to shale. The sand has constant $Q_P = Q_S = 10$. The correlation lengths of the random field for perturbing Q and elastic parameters are the same. The rms values are 4% for elastic parameters and 25% for Q .

modeling reflections in AVO analysis. We investigate the ability of the thin-slab method for handling elastic thin-bed reflections. The thin-bed used is 5 m thick, located at a depth of 1500 m and is filled with an oil sand of porosity 20%. The predominant wavelength is 125 m, being 25 times greater than the thickness of the thin-bed. The surrounding medium is shale. Source and receivers are on the top of the model. For the calculation using the thin-slab method, the grid spacing in the horizontal direction is 10 m and in the vertical direction is 10 m for the background area and 0.5 m for the thin-bed. Figure 18 displays reflection seismograms calculated by the reflectivity method (a) and thin-slab method (b). NMO corrections are applied to the top of the sand and reduced time is used in showing seismograms. In Fig. 18, A represents reflections from the top of the model, B gives reflections from both the top and base but no converted waves are involved, C includes all waves (exact solution) and D gives reflections from both the top and base including both single-leg and double-leg converted shear waves. Comparing A and B, we see the big difference between AVOs of a single impedance interface and two closely located impedance interfaces. B is equivalent to primary-only reflections. Comparing B with C and D, we see the important effect of locally converted shear waves that can alter amplitude variation with offset. Comparing C and D, only small differences exist at large offsets. In this case, the effect of multiples is negligible.

For a homogeneous thin-slab under small-angle approximation, equation for one-return reflection can be simplified into (Wu, 1996; Xie and Wu, 2001)

$$\mathbf{U}^P(\mathbf{K}_T) = -ik_\alpha \Delta z \frac{\delta Z_\alpha}{Z_\alpha} \mathbf{u}_\alpha^0(\mathbf{K}_T),$$

where Z_α is the P-wave impedance. The above equation presents not only the amplitude of the thin-bed reflection but also the change in wavelet. Therefore,

FIG. 18. Reflection seismograms for an oil sand model. The oil sand is 5 m thick and at depth of 1500 m. (a) By reflectivity method, (b) by thin-slab method. In the figure, A represents reflections from the top of the model (thick layer), B, reflections from both top and base but no converted waves included, C, all waves included (exact solution), and D, reflections from both top and base including single-leg and double-leg converted shear waves.

the thin-slab propagators represent the response of an elastic heterogeneous thin-layer. Since the choice of Δz or step interval in wavefield extrapolation is flexible and can vary according to local heterogeneities, the thin-slab method can naturally handle arbitrarily thin layers.

6.5. AVO Response in Laterally Varying Media

In this section we use a simple model containing both a truncated salt layer and a thin gas sand, which is located 200 m below the salt layer (Fig. 19a), to examine the AVO response using the thin-slab method.

The parameters for the gas sand are given in Table 1 and the corresponding porosity is 20%. The parameters for salt are $V_p = 4.48$ km/s, $V_s = 2.594$ km/s, $\rho = 2.1$ g/cm³. The thicknesses of the salt layers are 20 m, 40 m and 80 m, respectively. The grid spacing is the same as that used in Fig. 18 except for inside the salt body, where a 0.1 m grid space is used. In Fig. 19, the maximum negative amplitudes of the thin-bed responses are picked and plotted versus offset. The solid curve represents the thin-bed (without salt) AVO. The AVO trend of a thin-bed is different from that shown in Fig. 12 (shale/gas, $\phi = 20$) where the

FIG. 19. (a) Model containing a truncated salt layer with thickness d and a thin gas sand layer below the salt, (b) angle-dependent reflections (AVO) of a thin gas layer with reflections from the salt layer of different thickness.

reflection coefficient increases with incident angles up to 45° or an offset of 3 km. The dotted, dashed and dotted-dashed curves correspond to salt layer thicknesses of 20 m, 40 m, and 80 m, respectively. We see that AVOs are altered due to the presence of the salt layer. The salt layer can cause diffraction, defocusing, conversion and transmission loss. The diffraction affects AVO by causing interference between diffracted waves from the left side of the salt layer and reflected waves from the thin-bed at receiver locations. This interference causes local variations in AVO measurements. The transmission loss affects all the reflections passing through the salt body and causes a systematic decrease in AVO.

Figure 20 shows the combined effects of a truncated salt layer and random heterogeneities on AVO. Heterogeneities are introduced into the model shown in Fig. 19a. The correlation lengths of the 2-D random field are 100 m in horizontal direction and 40 m in vertical direction. The rms fluctuations used are 1%, 2%, and 3%, respectively. The four panels in Fig. 20 correspond to the cases of zero-salt, 20 m, 40 m and 80 m-thick salt (all labeled in the figure). The case of zero-salt shows the effect from heterogeneities alone. The focusing and defocusing of the spatially correlated heterogeneities produce the local variation in reflected amplitudes versus offset which becomes significant for sedimentary models with weak reflections. Interpretation of AVO observations based on homogeneous elastic models will therefore bias from actual properties of the target. The frequency and amplitude variations of reflections are closely related to the source spectrum and the statistical properties of velocity perturbations, although the overall AVO trends are still controlled by the target properties and overburden structures. To improve AVO analysis in sedimentary rocks, it is necessary to have

FIG. 20. AVOs of the thin gas layer with the combined effects of lateral structure variation and random heterogeneities.

sufficient acquisition apertures and take into account of the overburden structures including random heterogeneities and thin-bed effects.

7. CONCLUSIONS

In this chapter renormalized MFSB (multiple-forescattering–single-backscattering) equations and the dual-domain expressions for scalar, acoustic and elastic waves are treated in a unified approach. The De Wolf approximation neglects the reverberations (internal multiples) inside, but can model all the forward scattering phenomena, such as focusing/defocusing, diffraction, refraction, interference, etc., and the primary reflections. The De Wolf approximation can be considered as the first order term in a De Wolf multiple scattering series. This single backscattering signal defined this way is the *primary reflections* from the *real medium*, and is different from the Born approximation where the incident field and the Green's function are both defined in the reference medium (a homogeneous medium). It is also different from the first term of the generalized Bremmer series, which uses a high-frequency asymptotic solution (a WKBJ-like solution) as the Green's function (in an equivalent reference medium).

Two versions of the one-return method (using MFSB approximation) are given: one is the wide-angle dual-domain thin-slab approximation; the other is the screen approximation. The latter involves a small-angle approximation for the wave-medium interaction. Q -factor from intrinsic attenuations has been incorporated into the algorithm by the introduction of complex velocities. The reflectivity method has also been incorporated to the thin-slab modeling to increase the efficiency.

The theory and methods are applied to the fast calculation of synthetic seismograms. For weak heterogeneities ($\pm 15\%$ perturbation), good agreement between the one-return method and finite difference simulations verifies the validity of the one-return approach. However, the one-return approach is about 2–3 orders of magnitude faster than the elastic FD algorithm. The method can be applied to the fast modeling of AVA responses for a complex reservoir with heterogeneous overburdens. The method can be applied to most of the real cases where the perturbations of P- and S-wave velocities are around or smaller than 30%. The influences of heterogeneous or random overburdens, thin-bedding, Q -variation, irregular salt layer, etc., can be studied using this modeling technique.

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